DOCUMENT RESUME

ED 482 955 SE 068 327

AUTHOR	Emery, David
TITLE	Proceedings of the Undergraduate Mathematics Teaching Conference (Sheffield, England, September 4-7, 2000).
PUB DATE	2000-00-00
NOTE	115p.; For individual papers, see SE 068 328-329.
AVAILABLE FROM	Web site: http://www.umtc.ac.uk.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	EDRS Price MF01/PC05 Plus Postage.
DESCRIPTORS	Higher Education; Mathematics Instruction; *Numeracy; *Teaching Methods; Undergraduate Study

ABSTRACT

The 26th Undergraduate Mathematics Teaching Conference took place in September of 2000 at Sheffield Hallam University. Major topics of the conference included what use might mathematics education research be to university mathematics teachers?, attracting students to mathematics, use of the internet in teaching mathematics, and supporting the professional development of mathematics lecturers. Papers and presentations given at the conference include: (1) "The Popularization of Mathematics" (Simon Singh); (2) "The Professional Development of One Teacher of University Mathematics: Students, Maths, and History" (Bob Burn); (3) "Geometry in a Contemporary SettingÑMaking Connections" (Christine Hill); (4) "Proof and Generic Examples in Number Theory" (Tim Rowland); (5) "Embedding Key Skills in Mathematics Degrees" (Ken Houston and Neil Challis); (6) "Enabling Access to Further Mathematics" (Richard Lissaman); (7) "The Influence of Research Findings on the Restructuring of Mathematics Courses at University College Chichester" (Christine Hill); and (8) ""Serving" Our Students Right!" (Patricia Egerton). Also included are reports of the working groups. (MVL)

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ED 482 955

UNDERGRADUATE MATHEMATICS TEACHING CONFERENCE 2000

Proceedings of the Twenty Sixth Conference

4th to 7th September 2000

What use might mathematics education research be to university mathematics teachers?

Attracting students to mathematics.

Use of the Internet in teaching mathematics.

Supporting the professional development of mathematics lecturers.

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Published by

Sheffield Hallam University Press

SE063327

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Introduction

The millenium year saw the twenty sixth Undergraduate Mathematics Teaching Conference take place at Sheffield Hallam University. This was the fourth occasion on which it has taken place in the congenial surroundings of the Collegiate Cresent Campus. This year the conference was attended by thirty delegates.

The conferences began in 1975 with the title "University Teaching Conferences" and were for many years hosted by Nottingham University. They were initiated by the London Mathematical Society with the support of the Joint Mathematical Council and others.

The motivation for them was the (then) planned replacement of "A" levels by "N" and "F" levels. The conferences were intended to discuss the repercussions of these, and other changes in schools, on university mathematics courses. Notwithstanding the fact that the planned changes to "A" levels never happened, the conferences have continued to take place every year up to the present, which strongly suggests that they meet a real need of the academic community.

The reasons for this are not hard to find. Amongst the many benefits of attendance are: the sharing of knowledge and experience, an opportunity to reflect upon one's own teaching, contributing to the literature through the proceedings, establishing and renewing contact with mathematicians in other institutions, continuing professional development and personal refreshment.

All the conferences have been working meetings at which the delegates discuss, in small groups, issues of current concern to those teaching undergraduate mathematics. There are (usually two) plenary speakers who share their experience and insights with the delegates. This year we have been fortunate in securing the services of Dr. Simon Singh, the distinguished writer and broadcaster on matters mathematical and scientific, and Prof. Bob Burn, lately professor of mathematics education, Agder University College, Kristiansand, Norway.

Next year the conference is moving to Birmingham. After the conference left its first home at Nottingham University it was envisaged that it would move on every three years. In fact it has remained at its present home, Sheffield Hallam for four years and so a move is overdue. Maybe Sheffield made us too comfortable!

Birmingham is an auspicious choice for several reasons. Firstly its central location and easy access by road and rail should make it easy for delegates to reach. Secondly the establishment of the Mathematics, Statistics and OR Division of the Teaching and Learning Support Network at Birmingham provides a happy synergy with the work of conference. Birmingham itself is a lively metropolis which has played a central role in British history. It will therefore provide a rich choice of conference outing venues. Finally the excellent conference facilities of Birmingham University and the

Mathematics department would be reason enough in themselves to choose this new venue.

The organising committee is presently in discussions with the Institute of Learning and Teaching exploring the possibility of the conference becoming a partner organisation in one of the Institute's Continuing Professional Development pilot projects. If delegates can obtain some form CDP credit from attendance this should enhance the attractiveness conference, especially to younger lecturers.

A further change planned for next year is the introduction of a new type of working brief. Normally the bulk of the group's work is completed by the end of the conference leaving only the writing up of a report to be finished later. What is envisioned here, however, is a more extended piece of joint research activity to be planned during the conference but carried out over a period of a year or more following the conference. The research plan would be written up immediately after the conference for inclusion in the conference proceedings and the actual research itself would be expected to be published in a suitable journal. The inclusion of this type of brief should widen the appeal of the conference to those who are keen to enhance their publication profile.

Although the numbers attending conference have declined since its inception the conference remains in good health. We cannot, however, afford to be complacent. Conference must evolve in such a way that it continues to meet the needs of its members.

I have enjoyed my time as chair of conference enormously and I would like to record my thanks to all those who have helped me in that happy task.

I am most endebted to Dr. Neil Challis who, as well as acting as treasurer, happily accepted the photocopying and distribution of all the printed material with which I bombarded him. Whenever I was unsure how to proceed I would consult Neil and he never failed to offer me sage advice. Finally: Neil efficiently organised most of the "nitty-gritty" details on the ground such as room bookings.

I must also mention Neil's secretary Ann Shepherd who handled all the applications and I am sure many other things of which I am blissfully unaware.

Special thanks must go to Prof. Ken Houston who invited me to speak to the Heads of Departments of Mathematical Sciences and who gave me much needed moral support.

I am grateful to Prof. Peter Saunders who made the original suggestion for the new type of brief mentioned above, at the Heads of Departments of Mathematical Sciences at Leeds, Easter 2000.

There were many occasions when I called upon Pam Bishop of the Learning and Teaching Support Network for much needed information and I was never disappointed. I also wish to register my thanks to her for organising the meeting which Neil Challis and I had with Prof. Rob Curtis in order to arrange the transfer of the conference to Birmingham.

The conference website has for several years now been maintained by Dr. Neil Gordon and I was, like my predecessors, able to rely upon his rare skills. Many thanks Neil.

During the time the conference has been at Sheffield Hallam the conference outing and dinner has been organised by Dr. John Stone. This year he did us proud with a visit to The National Tramway Museum at Crich in Derbyshire followed by a spendid meal at the Peacock.

Dexter Booth , the Chair of Conference in 1999, supplied me with advice, deputised for me when I was on holiday and assisted me in my negotiations with the Institute of Learning and Teaching. For all this, and more, many thanks Dexter.

Finally I would like to express my thanks to all the members of the organising committee for their advice and support. In particular I wish the incoming chair Dr. Patricia Egerton every success with the twenty seventh conference and I trust she will enjoy herself as much as I did.

David Emery

David John Emery

Chair of UMTC 2000

The Plenary Sessions

The Popularisation of Mathematics

Dr. Simon Singh

The distinguished writer
and broadcaster on matters mathematical and scientific.

Abstract

Dr. Singh offers an explanation for the success of his book and the BBC documentary about Fermat's Last Theorem. He also describes his approach to popularising such an abstract subject, and the differences between the objectives of the science writer compared to the television documentary maker. The aim is also to give mathematics lecturers an insight into how the media view the popularisation of science and what their motives are.

Introduction

Mathematics is a notoriously difficult subject for the media to handle - most journalists are frightened of the subject and most of the audience would rather watch something else. However, a book and a TV documentary which have dealt with Fermat's Last Theorem and the proof of Andrew Wiles have both been well-received. The main reason for this is that the tale of Last Theorem is arguably the greatest science story of the decade.

It has all the elements of a Hollywood blockbuster. There is a seventeenth century genius who claims to solve an impossible problem ... but he never tells anybody his answer. He dies, leaving a tantalising note which challenges the rest of the world to match his proof. For the next three centuries the world's most brilliant minds attempt rediscover the answer, but all attempts end in failure. Even a million dollar prize fails to inspire a proof. Then, in 1963, a ten year old boy stumbles upon the problem. Undaunted he promises to devote the rest of his life to proving the Last Theorem. His obsession remains with him for decades until in 1986 a potential proof begins to take shape in his mind. He locks himself away, tells nobody of his idea, and spends the next seven years working on the calculation of the century. In 1993 he makes an announcement to the world, and is proclaimed the greatest mathematician of the age. He appears on CNN and on the front page of the New York Times. Then an error is discovered! Like the Terminator monster, the problem comes back to life and attacks our hero. Then, just as Wiles seems defeated, he announces that he has slain Fermat's Last Theorem. The centuries-old problem is dead.

The story does contain many beautiful mathematical ideas, but it is the power of the story which has captivated the general public. The challenge for the TV producer or the writer is to exploit this story in order to convince the public that mathematics is an exciting subject. For both the documentary and the book I will explain the commissioning process, their objectives, how they were made, and their impact on the public. I will conclude with a summary of the differences between the two media. Please note that this article represents my personal view of documentary film-making and writing, and other film-makers and authors may disagree.

The Television Documentary

The BBC has a science department devoted to making science programmes for the general public. The series which deals with science in most depth is "Horizon", which consists of over twenty hour long documentaries each year. Although the subject matter is science, the documentaries are not technical and are intended to reach out to an audience who may have little or no background in science, i.e., the documentaries must have a popular appeal.

As soon as Wiles's proof hit the headlines in 1993 the TV commissioners asked themselves a set of questions:

* Would a mathematics story help the balance of the series?
  Yes. Horizon has covered very few mathematical stories over the years.

* Is there a good story ?
  Yes. The human story is wonderful.

* Is it a significant achievement?
  Yes. This is one of the greatest mathematical breakthroughs of the century.

* Are the scientists involved eloquent?
  Yes. They are willing to tell their stories with passion.

* Are there good pictures ?
  No. There are no telescopes, laboratories, rockets, microscopes, etc.

* Does it have an broad appeal?
  No. People hate mathematics. They love dinosaurs and volcanoes.

* Is the science comprehensible?
  No. The proof is very difficult to explain.

Despite the three "No" votes, the decision was made to go ahead with the programme. However there was a severe problem - an error had been discovered in Wiles's proof! Fortunately in 1994 the error was corrected and work began on the programme in 1995. By this time John Lynch, who had originally proposed the TV project, had been promoted to Series Editor of "Horizon" and was too busy to make the programme on his own. Hence, he invited me to work alongside him.

The objective of the programme was not to explain the proof or give a definitive account, but rather to give an insight into Wiles's work and concentrate on what motivates mathematicians. It is impossible to explain anything in great detail on television because it is a 'linear' medium, i.e., the viewer has one chance to take in an idea, they cannot reread a page as they can when studying a book. The most that a TV producer can hope to do is to inspire the audience to want to find out more and enthuse them enough to read a book. It is also worth noting that television is a form of broadcasting, not 'narrowcasting', and our aim is to reach as wide an audience as possible. The main aim for a science film maker is to have non-scientists watch and enjoy a programme about a subject they previously knew nothing about.

To achieve the objective, the programme was structured around the human story of Wiles, which would attract a wide audience and stop them from switching to another channel halfway through. In particular, emphasis was put on giving the documentary a strong opening sequence - this showed Wiles being overcome with emotion, and I would defy anybody to switch off have watched this scene. The mathematical explanations were kept to a minimum and simplified to the extreme. The worst sin for a film-maker is to confuse the audience, who will immediately become frustrated and switch off. Initially the intention was to include significant amounts of history in the programme, but this was rapidly lost in favour of the modern story. It is far more interesting to see Wiles and Shimura giving first-hand accounts of their work than to have a faceless narrator giving a second-hand account of long dead mathematicians.

It takes approximately 5 months to complete a documentary. This involves extensive interviews with mathematicians, (4 weeks), scouting trips (2 weeks), preparation of a shooting script (2 weeks), filming (4 weeks), editing and postproduction (8 weeks). It is worth noting that at every stage the programme changes significantly, with many of the ideas in the shooting script (e.g., history) being dropped, and some sequences appearing apparently from nowhere during the editing process.

The programme was first broadcast on BBC 2 in January 1996, and attracted an audience of 1.8 million viewers, typical for an episode of "Horizon". The reaction from critics and viewers was almost entirely positive, with reviewers praising the unlikely mix of mathematics and emotion. It is also worth noting that mathematicians, including those that were involved, also enjoyed the documentary. The programme has since been broadcast as part of the "Nova" series in America (PBS-WGBH), in several European countries and on the BBC world satellite service. It has also won several awards, including the Prix Italia award for best documentary. I hope that part of the programme's success is due to the production and direction, but at the same time the eloquence and passion of the interviewees was vital, and furthermore a rich plot provided a strong foundation from which to build.

The Book

The objective of the book was similar to that of the documentary. The intention was to write a book for the layperson, rather than to provide a definitive account for the expert. It is worth noting that many scientists will consider themselves a layperson with respect to "Fermat's Last Theorem" - a professor of biology might know very little about number theory. In other words, my assumption was that my reader would be intelligent and curious, but not a mathematician.

Having worked in television for six years, my approach to writing was strongly influenced by my approach to making programmes. I was keen to maintain a strong narrative, I wanted to build characters, and I wanted to keep explanations to a level which would be accessible to a broad audience. In 1997 two other BBC TV producers wrote popular science books ("Mind Reading" by Sanjida O'Connell and "Feminization of Nature" by Deborah Cadbury) and having spoken to both authors, they also feel that their writing is influenced by their documentary film-making experiences.

Although my television background influenced my writing, there are significant differences between the book and the programme. First, the book consists of roughly 100,000 words, whereas the programme contained less than 10,000 words (interviews and narration). With ten times more material, it is possible to discuss points in more detail and include topics omitted from the TV programme. For example, the documentary covers the years 1700-1900 in 30 seconds, whereas the book devotes 100 pages to this period of history.

As well as history, the additional material in the book contains mathematical explanations which are far deeper than those in the programme, and yet still trivial relative to a serious mathematical text book. The extra history and the extra mathematics complement each other well, because by taking a strongly historical perspective on Fermat's Last Theorem it is possible to begin with fairly elementary mathematical ideas and build gradually on these. If the book had been firmly rooted in the twentieth century, then it would have been difficult to explain very much mathematics because the ideas are more complicated and there would have been very little to help the reader build up to these concepts.

With such a long history, so many mathematical ideas and so many good anecdotes to choose from, one of the major issues was which to include and which to omit. My criteria for inclusion were rather crude. First, if an idea or anecdote was essential, then clearly it had to be included. After that, ideas and anecdotes were included if they were interesting and easy to explain - even if ideas were not directly relevant but were still interesting and easy to grasp, I would still include them if I could find a way to weave them in to the story. Ideas which were difficult to explain and dull were not included.

The book took one year to write, working mainly at weekends and evenings. The response from readers and critics matched the reaction to the documentary. Once again, I hope that this is in part because of my writing, but I also acknowledge that Wiles and

Fermat's Last Theorem have given us a uniquely powerful scientific story. The book reached No. 1 in the British best-seller list (the first maths book to achieve this) and also reached the top ten in Italy and Germany. Although one should not judge a book by its cover, the design of the book was particularly eye-catching, and made it look more like a novel than a science book. I am sure that this attracted people who would otherwise shudder at the thought of picking up a maths book. Without a good publisher, effective marketing and support from the bookshops the book could have gone by unnoticed.

Conclusion

It is particularly difficult to get the public interested in mathematics, yet the success of both the book and documentary about Fermat's Last Theorem demonstrate that if there is a compelling narrative then readers and viewers can become captivated. Many mathematical breakthroughs will not be accompanied by such a powerful human drama, but I would advise those trying to popularise the subject to accentuate any drama that does exist. This does not mean ignoring the mathematics, it merely involves painting an image around the mathematics.

Finally, I think that it is worth reiterating three key differences between popularisation through the medium of television and a book. First, the documentary reached an audience in Britain of millions, whereas the book has only been read by tens of thousands. Hence, to really popularise mathematics, television is a more effective medium. Second, TV viewers are far more fickle than readers. Viewers have a remote control and have invested nothing in the programme, whereas readers are far more patient and committed, having paid money and made a concerted choice to read the book. This extra level of commitment means that an author can challenge and inform the reader much more than the viewer.

Thirdly, the length of the book allows the author to discuss aspects which are inevitably squeezed out of the documentary. The book is approximately 100,000 words, whereas the documentary is only 10,000 words. However, it is often said that a picture is worth a thousand words, and the programme is broadcast at 25 pictures per second for almost an hour, which means that it is equivalent to roughly 100,000,000 words! The opening sequence of the film shows Wiles recalling his breakthrough and being overcome with emotion - for me, this validates the calculation.

The professional development of one teacher of university mathematics: students, maths and history

Prof. Bob Burn

Lately professor of mathematics education,
Agder University College, Kristiansand, Norway

Retirement is a strange experience. It brings freedom and uncertainty about the future, but as your invitation has made me realise, it also provides a chance to reflect, to be aware of what one has become aware; to look back and to see what growth has been like.

In 1996 you invited me to UMTC and then I began by saying that there were no theorems in Mathematics Education. That is still true. But the last 15 years has seen a repeated call for theories. I am suspicious. Theories are surely there to explain and predict things. It is all too easy to develop theories when there are no facts to explain. In Mathematics Education we are extraordinarily short of facts. I don't mean by that that we are short of experience. I mean that most of our experience is so particular: a particular class, particular teacher, particular pupils, particular subject - so full of context, that our beliefs about what is generalisable are largely hunch. Each teacher generates her own conventions; each class new questions.

There are of course huge statistical data sets - TIMSS, the Cassels project, before that SIMSS, and in England, before that, CSMS. But when it comes to prescribing what successful teaching is like, this kind of research is no more helpful than any other.

So, what do we have: just anecdotes - the scorn of every theoretical researcher. Don't scorn anecdotes. Much good published research in education consists of organised or quantified anecdotes. A good anecdote can be representative; a story that you can return to again and again; a story against which you can match new experiences; a story that gives you a model which enables you to discuss education with a colleague. But you need your own representative stories. I am going to tell you some of mine. Most of them will present you with a tension, a dilemma on both of whose horns you must sit - ouch, a paradox that can take root inside you.

Identity

I began teaching in school in 1957. I was 23. The discomforts of that first year, which are still with me, centred on a fifth form O level class. I taught them five times a week and prepared for each lesson as if for a fight. Who was in charge? That was the contest. Other teachers have told me of their initial problems - refusing to give lectures until students had apologised for their insults. What to do when the apology never comes? Time lets most of us become secure enough in our teacher identity for these problems to diminish, provided we keep working at relevant subject knowledge and keep discussing teaching and learning with our colleagues. I mention this, not because this identity problem can be overcome by an appropriate inservice course or by reading, but because we older staff may need to offer a sympathetic ear to a new lecturer reminding them that some beginners' problems are universal, that it is maths we want the students to love, not us, and that honesty about one's ignorance is part of our integrity in the eyes of students.

Intentions

In 1961 I moved to a college in India where the B.Sc. course I was teaching bore a close similarity to the double maths course with scholarship papers that I had studied in the sixth form. (I should say that things have moved on a lot since I was there and the courses are now more modern and more advanced.) The identity problems I had known in England were not apparent until 1963, when to my complete astonishment, one of my classes struck and 90% of the students refused to be taught by me. I had been expounding the syllabus, setting problems from the textbook normally used in the college to be done as homework. My expectation was that the exposition and the exercises would generate the understanding with which students would successfully tackle the questions on the examination. Unhappily, only two or three of the students were completing the homework exercises. As a result of this strike, the normal teaching pattern was explained to me. The examination questions were highly predictable. A 'good' lecturer was expected to offer, say, twenty possible questions, word for word, and with full answers, written on the blackboard, for students to copy down and memorise. Out of these twenty, in a good year, four would appear on the exam paper out of a total of six. Competence on four questions would guarantee a first class degree. Passing the exam and getting a degree were necessary for a salaried job. The actual mathematics was of almost no importance for the future of any of the students.

So there was a clash of intentions. I had been seeking for competence and confidence through understanding mathematics. Students just wanted the degree and did not believe that my intentions were either necessary or sufficient for this purpose.
In this difficult situation I *had* to recognise student priorities. I went back to the standard college texts to try to understand why they had not generated the understanding I expected, and found not only a poor sequencing of results, but a quite chaotic sequencing of exercises. I had been brought up on Durell's *Modern Geometry* and *Projective Geometry*, books with pages of exercises with each one leading naturally on to the next. Without such constructive sets of exercises no wonder the students did not have the confidence to tackle questions on their own. Comforted and stimulated by the journals I was getting from Britain from the *Mathematical Association* and the *Association for Teaching Mathematics* I tried rewriting all the geometrical mathematics in the college. When the students knew I was working for their exam success they were willing to try the path of understanding which I was devising for them. It was this ghastly experience which made me turn to the journals for teachers, and when I could, get to conferences for maths teachers.

Questions

In 1965 I returned from India for a break to Britain, charged by the college principal with attending advanced courses so that he could claim my qualifications in applying for permission to run an M.Sc. course in the college. I was nervous about this, having suffered badly under the Cambridge Mathematics Tripos in 1953-55. But London University started a flexible post-graduate programme with M.Sc., M.Phil. and Ph.D that year on which I could start without much confidence. The experience was transforming. Mathematics changed from being something I learnt to something I did. Yes, we must keep learning, but it is the *questions we generate* which determine future possibilities. The flexibility of the London system worked to my advantage and I returned to India in 1967 with a Ph.D. thesis written and an M.Sc. course to teach, with smaller group size. The priority of *questions* stayed with me. I began to expect each class to begin with students *asking* about the homework they had tried. With one course there was no textbook which covered the course. So I wrote something equivalent for the students and then expected them both to read the text and solve problems before the class took place. One day there were no student questions. I said that if they had done any work, they would have questions, so I would wait for them. We sat in silence for an hour! Never again. From then on every class began with student questions. For this M.Sc. the college had discretion in awarding 40% of the marks. So we decided to give 10 marks in each course for the questions students asked. They scored 10 if they asked good questions. 5 if they asked bad questions and 0 if they asked no questions. You may be surprised to know that we never had difficulty deciding whether questions were good or bad!

Piaget

In 1971, having qualified Indian colleagues to run the M.Sc. course in India, I returned to Britain and accepted a post in teacher training at Homerton College, Cambridge. One of the things every primary teacher had to do in those days was to repeat some of Piaget's experiments on children of 5 or 6. I remember one in which a large glass jar was half full of water, with a stopper on. The glass jar was put into a brown paper bag and then laid on the table on its side. The children were asked to draw what the water would be like on an outline of the jar which they had been given. Some drew a maelstrom; some drew a water surface at 45° to the horizontal. The possibilities were amazing. The first conclusion was that we did not seem to know much about what children thought. The more worrying fact was that even if they had seen a jar on its side half full of water in the previous hour, it did not make much difference to what they drew. And that suggests that what we think is teaching may not be teaching. Of course sooner or later all children draw the water correctly in the jar, so there is no need to get chewed up about this particular example - unless it is representative.

I remember in Cambridge in 1953-55; I never missed a lecture; I took down all the notes the lecturers offered. I checked and corrected their logic afterwards. And I had no idea what the lectures were about. I was like the child who knew all about the upright jar and the brown paper bag, but had no idea what the horizontal jar was like. I had been taught that logic was both necessary and sufficient for mathematics. It *is* necessary, but it is *not* sufficient. The meaning was missing.

Investigations

During the 1960s members of the *Association of Teachers of Mathematics* had been stimulated by the papers of Imre Lakatos, which were later published (in 1976) under the title *Proofs and Refutations* by the Cambridge University Press, to consider the processes by which mathematics is made. At the same time College of Education courses had been lengthened from two to three years. The professional association for those colleges (*A.T.C.D.E.*) published advice about the content of these new courses, and the paper for mathematics was based on courses which had been developed at Clifton College, Nottingham and Derby. The paper differentiated between 'the teacher's mathematics' and 'the student's mathematics'.

The teacher was for the student, a window to mathematics across the world and back through the centuries; the student's main line of communication to the mathematics of other people. The student's mathematics was what she did for herself. For 'the student's mathematics' the college would suggest areas of enquiry beside or near the content of the standard courses. Not the standard exercises that take half an hour or less, but areas, not discussed in the literature, in which even the questions might need to be formulated, but which would need 10 to 20 hours for a good payoff.

I had heard tell of this style of work in the early sixties, but it was not until I did my Ph.D. that I began to understand the language of pattern-recognition, conjecture, counter-example and so on. It linked naturally to my experience of question-formulation which the research experience had made important, and once one began to see examples of it, it became obvious that research-like enquiry might be engaged in at any level of knowledge. There is nothing *necessary* about putting a dividing line between learning (first) and sustained enquiry (afterwards) at the *end* of a three year undergraduate course.

In 1974 I became Head of Maths at Homerton and after some discussion within the department, we began to offer our main maths students the opportunity for sustained enquiry during their first year in the hope that this kind of work might come to pervade all our courses. The experience was quite new for both students and staff. It was a way of bypassing the problems that Piaget's research had thrown up. Students would only write about what they wanted to write about, and therefore what they knew.
Because, in this style of working, as teachers we were almost entirely dependent on what students could and would do for themselves, this course (though it occupied less than ten percent of a year's work) had as transforming effect on our teaching as the Ph.D. experience had had on my attitude to mathematics. It showed us concept-formation happening in students because they needed the concepts. We wanted proofs, of course, but because we wanted much more than that, it put logic in a special functional perspective. This table indicates how our view of mathematics was enriched, and consequently how our view of mathematics teaching was modified.

Mathematics

process	product
heuristic	deduction
meaning	definition
guess, conjecture	theorem
examples	logic
plausible reasoning	convincing
test	proof
counter-example	contradiction
specialise	generalise

If all of this is mathematics, is one column the cake and the other the icing?

If so, which is which?

Knowing mathematics is both process and product; intuition and rigour.

Not knowing- From intuition to rigour

The syllabus and final exams for maths students in Homerton were under the control of the Cambridge Faculty Board of Mathematics. In 1978 there were plans to allow an additional optional subject paper for final year students, but for which no extra staffing allowance would be available. The college department was sure that it wanted investigations in this slot, and gathered supporting documents for such provision from other universities, especially Southampton. The request was turned down by the Faculty Board, for non-mathematical reasons. Those who had proposed investigations at the Faculty Board came back with a clearer set of parameters for what would be acceptable, but asked us searchingly, what do you really want? It was not an easy question to answer - but we said - we really want a course that the students can do by solving problems. We wanted to avoid the usual content based course in which mathematical necessities overrode the question of whether students could do the problems or understand the concepts. We were told - so long as there is a conventional exam at the end, there will be no problem with that. What subject would you like? We tossed various ideas about - topology, graph theory - without conviction. Then Frank Adams suggested Number Theory, because, he said you can put Hardy and Wright on the reading list, and then no one will say it is not a respectable course. In the college, we responded favourably to Adams' suggestion because, given a choice, most students preferred number theoretic investigations. No one in the college reckoned they knew enough number theory to propose a course, so I volunteered to construct one provided the University maths department gave me a contact to talk to. One afternoon I spent two hours with a research student (S.J.Paterson) who was finishing his thesis. He described an overall stucture for a course which he believed students could complete by solving problems. I made notes, and spent the next two years working out what those notes meant. Some years before, I remember Bill Brookes at an *A.T.M.* conference recommending that when we were stuck with a piece of mathematics, to keep notes of what happened during the stuck time, because after understanding was achieved the difficulties evaporated and one forgot what the problem had been. But that is *just* what the teacher must not forget.

I was stuck with number theory! But I kept a note book as I read and struggled with different concepts, different books and a range of maths problems. Time and again, it was particular computations which clarified theorems, clarified proofs and gave meaning to definitions. The 'stuck' notebook which I had kept was converted into 800 problems which defined the course *A pathway into number theory*. At the very heart of the process was the keeping of a record of how I had gone from not knowing to knowing, and using *that* as a path into the subject. Time and again, the difficulties were not logical but in the generation of meaning. Pólya made this distinction clearly and helpfully in his two books on *Mathematics and plausible reasoning*.

A review of this *Pathway* for the *THES* said that such a book on *number theory* was unnecessary as it was obvious that number theory could be studied this way, but that such a book on analysis would be worth publishing. My heart sank when I read this as analysis seemed to be so logic-ridden and counter-intuitive. But some years later, when I identified areas of my own ignorance in analysis, I tried the same device that Bill Brookes had recommended and found the consequent worksheets accessible to students in the way that the number theory had been. An analysis of the previous fifteen years of exam papers in Real Analysis revealed, to my surprise, that knowledge of about 50 examples of sequences, series and functions gave access to most of these exam questions, and so the beginnings of a pathway to analysis took shape.

Computer software

The 1980s also saw the arrival of microcomputers on the university maths scene. Perhaps these have been best used by departments of statistics for projects and by the calculus reform movement in the USA, and you will know other fine examples. Computers seem to be ideal for exploring the three forms of calculus information: numbers, graphs and algebra. I was involved in the design of geometric software, and developed student projects using LOGO and DERIVE. The software is doing its educational job when students are grappling with the subject directly and lectures are no longer amongst the obstacles with which students must contend. Students can become the victims of staff enthusiasms for software, unless staff recognise the hurdle which each piece of new software puts up and also recognise the time needed to get over the hurdle. Confident staff easily forget the difficulties of beginners.

History

I hope you will have noticed the dialectic in what I have been saying: learning more about *students* - listening better - and learning more about *mathematics* - how it is made, and what forms it takes. Each bounces back to the other. Professional development for me has been in the bouncing. And now, the latest step, particularly appropriate for the elderly(?!) - history. Twenty years ago, I had a vague sense that teachers had a moral obligation to know the history of what they were teaching. The first time I tried to implement that obligation was with *A pathway to number theory*, for which L.E.Dickson's encyclopaedic three volume *History of Number Theory* contains all the answers. There I got my first surprise. For I found that a certain historical development in the theory of quadratic forms exactly matched a pedagogical trick I had devised for my own understanding. A more interesting and substantial example was my struggling with the boredom on the face of submissive students on being presented with the group axioms. One can make up metaphors to try to persuade students that axioms are "a good thing", and cajole students into working with them, but the problem of making axioms purposeful was only solved by working first with symmetries, permutations and modular arithmetics and then summarising their common properties with the group axioms. Then deductions from the group axioms could be seen to have the power of multiple applications. In implementing this sequence (from permutations and symmetry to abstract groups) in *Groups: a path to geometry*, I realised that this was the way group theory developed historically.

The group axioms were formulated in the 1880s, but the study of permutation groups started with Lagrange in the 1780s and intensified with Galois and Cauchy in the first half of the 19th century. The study of symmetry groups started with Euler, took some big steps forward in the 1840s and was intensively developed by Jordan in the 1860s. Modular arithmetics were well understood before group axioms were formulated. So there really was something unnatural about starting group theory with axioms. It was chastening to recognise that Newton, Leibniz, Euler, Cauchy and Riemann did their mathematics without axioms for the number system, and it was even more chastening to find out that from the time of Weierstrass, for some thirty years, textbooks on real analysis offered at most one axiom, an axiom of completeness.

Of course there is nothing automatic about the relationship between individual development and historical development. For example, there is a profound mismatch in the development of negative numbers. But where the psychological research is thin, history offers a kind of longitudinal study of an actual development which may give clues for modern teaching; a study of how mathematicians have gone from not-knowing to knowing. The theory of integration is a prime example where historical development and intuitive development go hand in hand. The notion of function is one that has been subjected to intense psychological research to no great effect. This I believe is because the researchers are mesmerised by the simplicity of the Bourbaki definition in 1939, and ignore the early nineteenth century development from formula to continuous graph to discontinuous graph. The simple Bourbaki definition embraces too much. History repeatedly confirms Pólya's notion that special cases are the source of general theories. One thinks of Cauchy's functions $\sqrt{x^2}$, sin $1/x$, exp$(-1/x^2)$. Special cases indeed give us a handle on both theorems and proofs. But I find the hardest nut to crack, as a pedagogue, is that of definitions. But history here, and I think especially of Lakatos' study of $V - E + F = 2$, show examples and counter-examples exposing the possibilities and leading to a redistribution of formulations between Definition, Theorem and Proof. Definitions do settle down and become conventional, but they do not start out that way.

We seek to initiate our students into the richness of mathematics as a human endeavour. The confidence and competence which they develop is our reward. Their joy and commitment is the cream on our cake.

Bibliography

A.T.C.D.E, 1967, *Teaching Mathematics - Main courses in Colleges of Education*
The teacher's mathematics and the student's mathematics.

Burn, R.P. 1982, 1996, *A pathway to number theory*. Cambridge

Burn, R.P. 1985, *Groups: a path to geometry*. Cambridge

Burn, R.P. 1992, 2000, *Numbers and Functions: steps into analysis*. Cambridge

Dickson, L.E. *History of Number Theory*, 3 vols. Chelsea, and now A.M.S.

Gavosto, E.A., Krantz, S.G., McCallum, W. (Eds) 1999, *Contemporary Issues in Mathematics Education*, Cambridge University Press.
A fine selection from a conference on undergraduate teaching held in Berkeley, California in 1996. Delayed gratification and what of proof?

Jaworski, B., Wood, T., Dawson, S. 1999, *Mathematics Teacher Education*. Falmer Press
A account of how inservice courses in different countries have influenced the way school teachers on those courses see their work.

Lakatos, I. 1976, *Proofs and Refutations*. Cambridge

Pólya, G. 1954, *Mathematics and plausible reasoning*, I and II. Princeton.
Where do the ideas come from?

Primus (Problems and Resources In Mathematics Undergraduate Studies)
http://www.dean.usma.edu/math/resource/pubs/primus/index
PRIMUS is the only journal I know entirely devoted to undergraduate mathematics teaching.

Tall, D. 1992, The transition to Advanced Mathematical Thinking: Functions, Infinity, Limits & Proof. *in* D.A.Grouws (Ed.), *Handbook of Research in Mathematics Teaching and Learning*, Macmillan for NCTM.
Some classically thorny concepts in university mathematics and the psychological research on them up to 1992.

Talum (Teaching and Learning Undergraduate Mathematics)
http://www.bham.ac.uk/ctimath/talum/newsletter
See articles on "Learning to teach", issues 5 and 7.

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The Presentations

'Geometry in a Contemporary Setting' - making connections

Christie Hill, University College Chicester

Introduction

I first met 'Geometry in a Contemporary Setting' as a student myself when it was one of the taught modules for the MA in Mathematics Education at Chichester. I thoroughly enjoyed the scope of the content and the way topics were set in a historical context and related to other disciplines, in particular art, architecture and natural science. My opportunity to teach the undergraduate version of the module arose five years later when a colleague was on sick leave during the second semester of 1999/2000. The aims, objectives, indicative content and assessment criteria were therefore already established and, whilst I will be describing some of the content, my focus here is the way in which 'Geometry in a Contemporary Setting' leads students to make connections at a number of levels, from the practical to the philosophical.

Students taking the module:

Course	*Number of students*	*Year of course*	*Length of module*
2-year BA(QTS)/PGCE	7	First	Half-module
4-year BA(QTS) primary	3	Second	Half-module
4-year BA(QTS) secondary	3	Second	Full-module
3-year BA	1	Second	Full-module

Module aims

"Geometry has a central place in the development of mathematics in all cultures; this module places that development in a modern context. Geometrical thinking, knowledge and understanding contribute strongly to the solution of mathematical problems and to wider mathematical and general understanding. Students are introduced to a range of themes within their geometrical heritage."

Module objectives

"At the conclusion of the module students should be able to:

1. appreciate the history of geometry and its place in mathematics; know selected geometries and hold a wider perspective on geometry; be able to identify geometric properties of artifacts and in nature;

2. have acquired skills in using drawing instruments, making models, using computer

packages and writing programs in a geometric context;

3. show familiarity with a range of geometric representations of objects, and thus be able to follow geometric arguments and to tackle geometric problems;

4. research an aspect of geometry at a deeper level, by reading widely, and by using source material in carrying out geometrical enquiries."

Assessments vary to take account of the different courses, but all include individual logs of studies undertaken and students are told that:

"Tutors will look for evidence of clarity and accessibility of presentation and organisation; extent of coverage of the various topics met in sessions; effective use of a range of resources; quality of the critical self-evaluation."

Module outline

The first six weeks of the module were followed by all students:

Week 1	Plane geometry 1	Polygons; tilings & tessellations (regular and semi-regular); symmetries; transformations.
Week 2	Solid geometry	Euler's theorem; angle deficiency; regular & semi-regular polyhedra; duals.
Week 3	Plane geometry 2	Greek constructions; four centres of triangles; circle theorems; use of Geometer's Sketchpad software.
Week 4	Cartesian geometry	Descartes (1637) - application of algebra to geometry.
Week 5	Geometry in art, design & nature	Proportion (Golden Ratio); perspective; visual illusions; Escher; Mondrian; spirals; fractals.
Week 6	Other geometries	The parallel postulate; history of non-Euclidean geometry; taxicab geometry.

Each module involves three hours of contact time and six hours of directed study per week so students are expected to consolidate classwork and extend work on topics of their choosing. The content of a mathematics module provides immediate learning objectives but the overall aim of courses at UCC is to set students thinking about the nature and processes of mathematics and to develop their own mathematical thinking.

Module sessions - a brief overview

The study of tessellations and polyhedra provides a very practical introduction to an area of geometry which has a long history but is new to most students and requires little prior knowledge. The obvious connections are the physical ones between the polygons which make up both tessellations and polyhedra. Then there are the visual connections to be found within tessellations and polyhedra which can be identified in terms of translations, rotations and reflections. Most students have already studied Graph Theory and can demonstrate the connection between polyhedra and Euler's theorem but, as we shall see later in more detail, investigations can reveal further numerical patterns and algebraic connections which require proof.

Prior knowledge does give students an advantage when Euclidean geometry is tackled in the third week but this is offset to some extent by introducing dynamic geometry software (Geometer's Sketchpad) as a modern form of 'ruler and compass' construction. Students seem to enjoy the versatility of this tool and the way in which rigorous constructions can be transformed to give a multitude of examples of a particular geometric property. Faulty constructions are soon revealed as they fall apart or otherwise distort when transformations are attempted so students quickly appreciate that geometrical connections between different elements of a construction need to be mathematically secure, - not simply 'look right'.

Cartesian geometry in week four allows students to make more connections, this time between geometry, algebra and co-ordinate geometry. Geometer's Sketchpad is again a useful tool, as is a graphical calculator. Some of our students find the derivation of algebraic formulae from Euclidean definitions difficult, but the graphical representation of algebraic functions provides accessible visual evidence of the connection between algebraic and geometrical definitions..

The geometry of art, design and nature is a very wide area and the contact time allows for only brief introductions to aspects which students might like to pursue including:

- the development and use of perspective in art, including classical pictures, Op Art, visual illusions, including 'impossible figures', and advertisements;
- proportion, particularly the Golden Ratio, and its significance in architecture, paintings both classical and modern (Mondrian) and nature (spiral shells, seed-heads, cones);

- fractals, - methods of generating fractal patterns and their occurrence in nature (ferns).

Identifying the geometry inherent in much of our everyday surroundings, and our preference for particular geometrical forms, demonstrates that mathematics is an integral aspect of 'life' and that connections are there to be discovered and explored.

The final session addressed the nature of other geometries, initially from an historical perspective. It is difficult for students to get a feel for another geometry when they usually sense their world as Euclidean, so taxicab geometry was introduced as a way of connecting the 'alien' world of non-Euclidean geometry to their own experience. As with earlier sessions it was the 'hands-on' work, of constructing taxicab 'circles' for example, which brought some understanding of the nature of this form of geometry.

Polyhedra

Polyhedra is a topic which I and colleagues have introduced to school children at a number of mathematics workshops using plastic polygons which clip together or polygon mats which can be glued (produced by *Polydron UK Ltd.* and *ATM* respectively). We wanted to extend the activity beyond simply following diagrams to build polyhedra, so our first objective was for children to predict the shapes and numbers of polygons which give rise to regular polyhedra using the fact that there must be at least three faces meeting at each vertex and the sum of angles at a vertex must be less than 360 degrees. This gives rise to a proof by exhaustion that the five Platonic Solids are the only regular polyhedra.

Then counting faces, edges and vertices for each of the Platonic solids leads to the conjecture:

$$f + v = 2 + e \Sigma d = 180(s-2)^\circ$$

whilst defining the angle deficiency, d, as the amount by which the sum of angles at a vertex falls short of 360 degrees allows the total angle deficiency to be calculated for each solid and leads to a second conjecture:

$$\Sigma d = 720^\circ$$

We did not attempt to prove these formulae when working with Key Stage 2 pupils but used them to check possible structures for the semi-regular Archimedean solids.

Warwick Evans has explained the method very clearly in an article published on the NRICH website (http://www.nrich.maths.org.uk/mathsf/journal/oct00/art2/index.html).

My aim with undergraduates was to enable them to discover and prove these connections for themselves. The students already knew Euler's theorem and could apply it to polyhedra but they needed to discover and prove that the total angle deficiency for a polyhedron is 720 degrees. The previous week's work on tessellations moved into 3-dimensions when students investigated what happened when the angle sum at vertices was less than 360 degrees, so they had already encountered angle deficiency. However their first inclination was to sum angles at vertices, not angle deficiencies, when tabulating data for various polyhedra, but the total angle deficiency conjecture was eventually made. This gave students the tools to investigate polyhedra during non-contact time and left the proof of the conjecture to complete in class.

Proving the conjecture

The first difficulty is to move from checking specific examples of polyhedra to a generalised situation. The first stage is to recognise that the shapes of polyhedral faces must be somehow shown to be immaterial to the proof. Looking at the three regular polyhedra with triangular faces can help here, for all three can be shown to result from the same proof, the numbers of faces is irrelevant. This should be the clue leading to the idea of triangulating the faces of polyhedra but I was unable to lead students to this without a further visual stimulus. Fortunately *Polydron* produces right angle triangles which clip together to form a standard square as used for constructing the cube. By constructing the cube from these triangles I was able to develop the idea that, given suitable triangles, all polyhedra could be constructed from triangles.

At this point the students had all the information necessary to construct the algebraic formula needed to prove the conjecture. They recognised that Euler's theorem and triangulation would be of use, but needed prompting to construct relationships between vertices and edges and faces and edges. Once these were in place we were able to progressively eliminate the variables until it was clear that the total angle deficiency of any polyhedron was 720 degrees.

This, to me, is an interesting proof for two reasons. In the first place we know when we start what the 'answer' will be, but the number of variables involved still makes the final result somehow surprising. Secondly it relies on very accessible geometrical ideas which are readily understood by students.

So far students have not made the connections necessary to construct the proof for themselves but I think it should be possible, given an appropriate context and approach.

I believe that level of success would give them an even greater understanding of the nature of proof, so if colleagues have ideas for making the proof more accessible please make them known.

Proof and generic examples in number theory

Tim Rowland, Homerton College, University of Cambridge

In this paper, I distinguish between the purposes of proof in terms of (i) community-of-practice certification, and (ii) enlightenment and explanation of inductive discoveries and standard theorems. I argue the case for wider acceptance of the appropriateness and validity of generic arguments for the second purpose, and for more attention to the deliberate deployment of generic examples as didactic tools.

Proof is central to mathematics, yet the purposes of proof need careful consideration in order to achieve some consensus on what might count as a proof in a given context. Reuben Hersh (1993) argues that the role of proof in the classroom is different from that in research. For research mathematicians, claims Hersh, the purpose of proof is the assurance of 'truth', but the situation in the classroom is very different.

> In the classroom, convincing is no problem. Students are all too easily convinced. Two special cases will do it. (Hersh, 1993, p. 396)

In the teaching context, the primary purpose of proof is to *explain*, to illuminate why something is the case rather than to be assured that it is the case. In the teaching and learning of number theory, proof by generic example (Pimm and Mason, 1984; Balacheff, 1988) can be effective for such explanatory purposes.

> The generic proof, although given in terms of a particular number, nowhere relies on any specific properties of that number. (Pimm and Mason, 1984, p. 284)
>
> A generic example is an actual example, but one *presented in such a way* as to bring out its intended role as the carrier of the general. (*op. cit.* p. 287, emphasis added)

The story (probably apocryphal, but see Polya, 1962, pp. 60-62 for one version) is told about the child C. F. Gauss, who astounded his village schoolmaster by his rapid calculation of the sum of the integers from 1 to 100. Whilst the other pupils performed laborious column addition, Gauss added 1 to 100, 2 to 99, 3 to 88, and so on, and finally computed fifty 101s with ease. The power of the story is that it offers the listener a means to add, say, the integers from 1 to 200. Gauss's method demonstrates, by generic example, that the sum of the first $2k$ positive integers is $k(2k+1)$. Nobody who could follow Gauss' method in the case $k=50$ could possibly doubt the general case. It is important to emphasise that it is not simply the fact that the proposition that the sum $1+2+3+ \ldots + 2k = k(2k+1)$ has been verified as true in the case $k=50$. It is the *manner* in which it is verified, the *form of presentation* of the confirmation.

> The generic example involves making explicit the reasons for the truth of an assertion by means of operations or transformations on an object that is not there in its own right, but as a characteristic representative of the class. (Balacheff, 1988, p. 219)

In this way the generic example serves not only to present a confirming instance of a proposition - which it certainly is - but to provide insight as to *why* the proposition holds true for that single instance. The transparent presentation of the example is such that analogy with other instances is readily achieved, and their truth is thereby made manifest. Ultimately the audience can conceive of no possible instance in which the analogy could not be achieved.

For some years, I taught a second year undergraduate course in number theory, covering the usual topics found in texts such as Baker (1984). Modular arithmetic arises early in the course, with the theorem that every prime number p has a primitive root (that is to say, the group $\{1, 2, 3, ..., p - 1\}$ under multiplication modulo p is cyclic). The standard general proof (see, for example, Baker, 1984, p. 23) is surprisingly indirect and overburdened with notational complexity. This latter factor is, in part, due to the fact that it has to deal with a double layer of generality: any prime p and any divisor d of $p - 1$. In recent years I have opted for generic exposition of the proof, by reference to the particular case $p=19$. To evaluate the pedagogic effectiveness of this strategy, students were asked to complete questionnaires which asked (*inter alia*):

- Does the explanation for $p=19$ convince you that $p=29$ has a primitive root?
- Does the explanation for $p=19$ convince you that every prime has a primitive root?

Some two-thirds of the students responded to both questions in the affirmative. Their comments were characterised by the student who wrote:

> It is easy to follow the logical progression of the proof for $p=19$ with any other prime in mind, and I can see no area of the proof which gives me any doubt that it wouldn't work for any prime.

I believe that learners of mathematics at all levels, including university students, should be assisted to perceive and value that which is generic in their own particular insights, explanations and arguments. The barrier between such a level of knowing and the writing of 'proper' proofs is then seen for what it is - a lack of fluency not with ideas, but with notation. I therefore urge that more conscious attention be given in mathematics teacher education – at all levels – to the deployment of generic examples, as didactic tools, for purposes of explanation and conviction.

Note: an extended version of this paper is to appear in a monograph on Number Theory (edited by Rina Zaskis and Stephen Campbell) under the auspices of the *Journal of Mathematical Behavior*

References

Baker, A. (1984) *A Concise Introduction to the Theory of Numbers*. Cambridge, Cambridge University Press.

Balacheff, N. (1988) 'Aspects of proof in pupils' practice of school mathematics'. In Pimm, D. (Ed.), *Mathematics, Teachers and Children*. London, Hodder & Stoughton, pp. 216-235.

Hersh, R. (1993) 'Proving is convincing and explaining'. *Educational Studies in Mathematics* **24**, pp. 389-399.

Mason, J. and Pimm, D. (1984) 'Generic examples: seeing the general in the particular'. *Educational Studies in Mathematics* **15**, pp. 277-289

Polya, G. (1962) *Mathematical Discovery, Volume I*. New York, John Wiley and Sons.

Embedding key skills in mathematics degrees

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Ken Houston, University of Ulster

It was discovered a long time ago (in CNAA days) that teaching "General Studies" through an "add-on" module was a waste of time. Students treated it as a joke and it added very little to the value of their higher education. It wasn't until the history, sociology and cultural aspects of their subject were included in mainstream modules, taught by mainstream subject teachers, did students begin to apply themselves seriously to a study of these aspects of their subject. It wasn't until students had something really interesting to talk and write about, did they begin to talk and write in a useful manner. The "embedding" principle must be applied today to the learning of Key Skills. Encouraging students to improve their reading, writing and talking skills must be in the context of their mainstream subject when they will have something that really interests them to read, write and talk about, or to collect and analyse data in relation to.

A rationale for including key skills development, if it is not immediately self-evident, comes from the Dearing report of 1997 and two recent surveys by the professional bodies SIAM in the USA and IMA in the UK.

The SIAM report (1998) contained data from a survey of PhD graduates working in industry. The report indicated that modelling, communication and teamwork skills together with a willingness to be flexible are important traits in employees. But the PhD graduates themselves indicated that they felt inadequately prepared to tackle diverse problems, to use communication effectively and at a variety of levels, or to work in teams.

The IMA was a principal player in the MathSkills project which carried out surveys of mathematicians in employment and of the employers of mathematics graduates (MathSkills, 1999). It was acknowledged that graduates had the desired subject specific knowledge - computer skills, basic mathematics, numerical methods, statistical methods and inference, modelling and calculus. But it was found that employers thought graduates to be lacking in presentation and communication skills, social skills, pragmatism and commercial awareness.

The Mathematics Department at the University of Ulster, when designing new courses in the mathematical sciences at degree and diploma levels, decided to take modelling as the theme, to support this through the teaching of appropriate mathematical, statistical and computing methods, and to embed key skills activity where appropriate. Thus, through group work and specifically designed tasks, students learn to develop their skills of reading, thinking, questioning, criticising, discussing, writing, speaking, explaining and listening. Some of the innovative methods employed are - using comprehension tests, peer tutoring, student lead seminars and a variety of presentational methods including the use of poster sessions.

The Mathematics degree at Sheffield Hallam, with an applied and modelling flavour, was similarly designed from the beginning to embed aspects of skills development in a coherent way through many parts of the course. Key features include: some group working at all levels, and some partly discursive assignments at all levels; use of a wide range of assessment methods including posters, oral presentations, written reports, and portfolios, as well as various types of examination; an optional but recommended industrial placement; and a substantial individual final year project which in the best cases allows the student to demonstrate the full range of skills.

A common conclusion from both institutions is that a strategy for embedding key skills will best succeed if it has the full support of senior university officers, staff believe in it and students appreciate its worth. The strategy employed at both universities is to require all courses at initial evaluation or at quinquennial review to demonstrate the ways in which key skills will be developed. Both universities recognise the need for, and support, staff development in this area.

This is a condensed version of a paper written by the same authors (Challis and Houston, 2000).

References

Challis, N. V. and Houston, S. K., 2000, 'Embedding Key Skills in the Mathematics Curriculum', *Capability*, to appear.

MathSkills, 13 May 1999, *The MathSkills Discipline Network*, <http://www.hull.ac.uk/mathskills/> (accessed 3 July 2000)

SIAM, 30 January 1998, *The SIAM Report on Mathematics in Industry*, <http://www.siam.org/mii/miihome.htm> (accessed 3 July 2000)

Enabling access to further mathematics

A mathematics in education and industry project sponsored by the Gatsby charitable foundation

Richard Lissaman, University of Warwick

What is the Nature of the Problem?

- The number of students taking Further Mathematics A level has dropped from about 15000 in the early eighties to about 5000 now.
- Many students who would benefit from Further Mathematics are missing out.
- Universities and employers value Further Mathematics qualifications but are unable to require them because it would discriminate against the majority of students who do not have the opportunity to study for them.

What are the reasons for the decline in the number of students taking Further Mathematics?

There are two major reasons for the decline:

- In most school sixth forms only a small handful of students will wish to study Further Mathematics. This makes prohibitively expensive to run.

Perhaps 20% of Maths A level students would benefit greatly from the opportunity to study Further Mathematics. In a typical school sixth form this is only two or three students.

- Many school Maths departments do not have staff available with the experience or confidence required to teach Further Mathematics.

This has become self-fulfilling. Few Maths teachers who have entered the profession in the past 10 years or so have studied Further Maths themselves and many are intimidated (unnecessarily) by it.

How can we address the problems?

The MEI "Enabling access to Further Mathematics" project is designed to show that Further Maths can be made available cheaply and effectively in all sixth forms via distance learning.

- The project is a 3-year pilot, funded by the Gatsby charitable foundation.

- Each student will have access to a fully resourced and interactive web site, textbooks, a mentor and a tutor.

- Tuition will be via e-mail, fax, telephone, video-conferencing, web forums and weekend seminars.

The scheme would also lend itself to use by teachers who may wish to 'brush up' on their own mathematics in order to begin teaching Further Maths themselves.

If you are interested in finding out more about the project or wish to become involved, please contract either

Charlie Stripp (Project Co-ordinator)
e-mail: charlie@mei-distance.com

Richard Lissaman (Director of Warwick lead centre)
e-mail: rml@maths.warwick.ac.uk

The influence of research findings on the re-structuring of mathematics courses at University College Chichester

Christine Hill, University College Chicester

A research project at University College Chichester entitled Flexible Learning Approaches in Mathematics in Higher Education (FLAMHE) contributed to the restructuring of mathematics courses when the College changed from a three term to a two semester academic year.

Mathematics education researchers had carried out case studies of students' experiences of mathematics courses during their first year as undergraduates at University College Chichester and five other universities. We also observed and analysed the teaching of particular courses at University College Chichester in terms of cognitive and affective issues as well as mathematical understanding.

This research had a direct influence in at least four areas when re-structuring for the two semester year took place:

- the departmental entry in the College prospectus;
- the content of modules, particularly those for first year undergraduates;
- the introduction of innovative teaching/learning approaches;
- the choice of assessment methods for modules.

Our research had shown that students wanted more information about the structure of courses and the choices open to them during their undergraduate studies. When did they have to decide between a BA/BSc and an MMath? Could they change from a 4-year secondary QTS course to a 3-year BA course? Would they be able to study Chaos theory? We changed our information format from text to tables to make the structure and decision points for courses clearer and included diagrammatic outlines of possible module content for each course. Prospective students at Open Days commented on the visual impact and clarity of these forms of presentation.

The wide variety in students' mathematical knowledge was already well known but case study students pointed out that the content of the A-level (or other) course taken was often more relevant than the overall grade achieved - some familiarity with a topic gave students a head-start over those to whom it was completely new. We wanted students to start with 'a level playing field' and this could be achieved if first year modules either addressed areas of mathematics familiar to all students or introduced topics which were completely new to all.

To some extent this approach was already in place. Two foundation mathematics modules focused on consolidating A-level mathematics and 'filling the gaps'. However students were expected to find the gaps in their own knowledge and it had become clear that they needed help to identify their own weaknesses because by definition 'you don't know what you don't know'. Particular weaknesses common to most students were therefore identified and are addressed in the two new courses, 'Core Mathematics' and 'Methods and Techniques'

During the first semester 'Number Theory' and 'Algorithms and Programming' provide topics unfamiliar to most students, whilst 'Problem Posing, Problem Posing' challenges their attitudes and beliefs about the nature of mathematics. In the second semester 'Graph Theory' is usually unfamiliar whilst 'Modelling' can be differentiated for varying levels of previous knowledge.

Case study students also drew our attention to the difference in teaching methods and expected attitudes to mathematics for A-level (and other FE) studies and undergraduate work. Their A-level work had usually been 'teacher-led' and many of them saw mathematics as a 'right or wrong' subject with rules and procedures which were to be learned but not necessarily understood. Support from teachers was usually readily available on site.

Mathematics teaching/learning at Chichester already used group work, projects, posters and individual and group presentations but further innovative approaches have been introduced to ease the transition to undergraduate work and encourage students to take more responsibility for their own learning. Students taking 'Applications of Calculus' in year two run a seminar for their assessment task, whilst in their final year students study 'Algebra' by a mixture of small group tutorials and peer tutoring, and again run a seminar for assessment.

In addition tutors began to use the College intranet as an integral part of some modules, providing information and tutorial support for students whether on-site or not.

All these innovations have also contributed to students' transferable skills profile.

Undergraduate assessment usually includes a number of formal examinations but mathematics departments also assess course work in various ways including summative and formative writing, formative tasks and student presentations, both oral and visual.

The variety of assessment methods and the range of transferable/key skills expected from and by our case study students led to an analysis of the relationship between module content, transferable skills and methods of assessment. The form of assessment for individual modules was then chosen to be both appropriate for the content and to maximise the development of transferable skills.

Tables listing course modules against either assessment method or transferable skills allowed tutors to see that both assessment and skills were distributed reasonably uniformly across each year of the course. This 'grid' method was very successful and taken up by other departments in the Science faculty at Chichester.

The influence of the FLAMHE project on mathematics courses at Chichester continued as tutors evaluated the effect of initial changes and made further modifications in subsequent years. The whole project represents a collaboration between mathematics teaching and research and evidence of action research on the part of tutors.

"Serving" our students right!

Patricia Egerton, University of Teesside

In this brief presentation my emphasis is on "Service Mathematics", hence my title. In universities many of us are involved with supporting the mathematics learning of students whose main subject area is not mathematics; I believe that as soon as we accept this role these students too become 'our students', deserving our best attention.

I am considering situations when we are asked to deal with students on degree courses whose entry requirements do not necessarily include a pass in Mathematics at A level, such as computer sciences, natural sciences, business, engineering, sport science, social sciences, and health. As part of my National Teaching Fellowship project, I am investigating how well the appropriate mathematical techniques and understanding are delivered to these students, who come to us with a wide range of prior mathematical achievement, from A level to a GCSE or an Access qualification. At present I am measuring student success by nothing more than a 'pass' in their maths assessments; I am not measuring their retention or flexibility or 'deep learning', just how well they pass the exams we set, the exams which form the internal basis of our judgement systems.

This project is still in its initial phase, so all I do here is give a flavour of some of the evidence from my pilot study. I have looked at a fairly large cohort of 'computing' students who have to study a module of 'Computing Mathematics' in their first semester, followed by either 'Graphical Mathematics' or 'Formal Methods' in their second. These modules are important since the programme contains 32 modules (out of 139) which have 'Computing Mathematics' as a prerequisite. I analyse the entry qualifications of the whole cohort and group them into three bands: 'passed Maths at A level', 'studied Maths or a cognate subject at A level', and 'no mathematics since GCSE'. I note each student's exam result in both first year modules, and group and graph the findings; only by undertaking this level of detail can I discover what is significant. The pass/fail proportions of the three bands of students in the three modules tell me (among other things) that there are significant differences in the delivery, assessment and/or perception of the modules. I hope to identify the nature of these differences by discussing teaching and learning styles with staff; then to initiate realistic sharing of good practice for the benefit of all students concerned in the future.

I am also working with the maths results of cohorts of engineering students, and shall then spread this investigation to other subject areas. There are several issues to be considered overall:

*is the content/pitch/support of service teaching offered fairly to all students?
*are staff sufficiently aware of the need to differentiate their material?
*are some students with Maths A level demotivated by repetition?
*are some students without Maths A level put under too much pressure?
*are there teaching styles to assist more students to succeed in their learning?
*are the results any different when the host department delivers its own 'service maths'?

I hope to be able to gather information from other institutions in UK and abroad, and I will appreciate any suggestions, or pointers to the future, examples of innovative practice, and offers of cooperation. I invite comments from all interested parties, and will respond to all communications!

Reports of the Working Groups

What use might mathematics education research be to university mathematics teachers?

John C. Appleby, University of Sunderland

Judith M. Ekins, Open University (scribe)

Christine Hill, University College Chichester

S. Ken Houston, University of Ulster (chair)

Paul Wilson, National University of Ireland, Galway

Introduction

In 1996 a conference on the future of mathematics education at research universities was held at the Mathematical Sciences Research Institute in Berkeley. The report of the working group on research mathematicians and research in mathematics education begins:

> *Mathematics education research is a field of inquiry into the nature of mathematical learning, as well as into the practice of mathematics teaching. It provides a foundation and methods for designing diverse teaching strategies and for studying their effects. The study of mathematical learning investigates the process by which students give meaning to and learn to employ mathematical ideas and practices, by making connection with and updating their prior knowledge and experience. Such investigations not only provide basic knowledge essential to the development of curricula and materials, but can significantly inform teaching practice as well.*
>
> *Gavosto et al 1999*

This definition provides a good introduction to the endeavours of this working group.

Schoenfeld (1991) describes three classes of mathematics education research:

1. Basic Research into Cognition – enquiry into the nature of thinking and learning process – not driven by any obvious application
2. Social and Epistemological Engineering research - social engineering of pedagogical contexts for epistemological purposes e.g. evaluating changes in curriculum design and delivery;
3. 'Product-Oriented' research which is the classic applied work of mathematics education, brought up to date with the addition of computer technology.

To which can be added

4. Surveys of current theory and practice.

There are a number of ways in which mathematics education research might be of use to mathematics teachers, for example:

A. They can learn from research studies of education researchers;
B. They can become 'teacher researchers',
C. They can collaborate with specialist education researchers and thus bring an inter-disciplinary approach to their research enquiry;
D. They can feel part of the community of mathematics educators.

There are an increasing number of studies in research classes 1 to 4, from which mathematics teachers can learn (A above): examples are given in Section 2. The idea, in B above, of being a 'teacher researcher' is widespread in education at all levels. This is a method, which can be widely used to inform and improve our teaching and is developed further in Section 3. Section 4 gives examples of being a 'teacher researcher' and being involved with teacher-specialist collaborations, from the group's own educational research experiences.

All research is about finding answers to questions. Section 5 discusses research questions, which the group considers worthy of further detailed research.

Examples of educational research.

This section looks at a selection of papers in the area of mathematics education research, which might be useful to mathematics teachers, as suggested in point A above. Whilst it is not claimed that these papers are a representative cross section of the area, they do give an idea of the breadth and variation, encompassing all four classes of mathematics education research, mentioned in the introduction. However several of the papers fall into more than one category.

Nardi(1999) The novice mathematician's inquiry about new concepts: bestowing meaning through ambivalent uses of geometrical metaphors

This research belongs to research class 1, in that it uses cognitively based theories to structure an analysis of the development of understanding through the use of geometrical metaphors. Her example of a 'Paradigmatical Episode' includes a step-by-step analysis of the teaching and learning it represents. This could give other mathematics teachers insights into the progress of such learning processes, including the pitfalls, and provide them with the incentive and method for investigating the development of understanding in their own students.

Tall and Vinner (1981) Concept Image and Concept Definition in Mathematics with particular reference to Limits and Continuity.

This paper also describes a research project in class 1, in that it attempts to explore the cognitive structures that the learner is using. The concept image of the title consists of everything in the learner's mind associated with a given concept. All of this may not be globally coherent and may include aspects, which are quite different from the formally taught concept definition. These differences may cause cognitive conflict. The authors report investigations they have carried out in the learning of limits and continuity as taught in secondary school and university. The paper may be of use to university mathematics teachers in that it gives a general theory of learning that helps explain some of the difficulties students may have with a particular topic. Thus it provides insights of use in developing a teaching strategy.

D'Inverno (1993) "On the success of a self-paced course"

This is a review of the evolution of the self-paced mathematics course for first year engineers at Southampton University over more than 20 years. It provides a longitudinal study. It is an example of research class 2 (social and epistemological engineering research) and could offer valuable information for teachers considering this form of teaching. It is essentially a descriptive and reflective report of an action research process.

Alibert (1988) Towards new customs in the classroom

This paper describes an experiment using the idea of classroom 'customs' - explicit or implicit rules that drive the working of the system and what teacher and students expect of each other. So it is in research class 2.

The basis for the research was the observation that many students' actions simply mimic the writing of the teacher, rather than being driven by a desire to 'control' meaning. For example they regard proof as a formal exercise rather than a functional tool.

New 'customs' were adopted at a French university over two years, with 100 first year students. 'Scientific debate' was used: students first trying to convince themselves and then other students. Conflicts and contradictions led to the need for proof. Evaluation was by observation and student questionnaire. Examples for debate were prepared in advance by a research team and included conjectures in analysis and a gravitation problem from mechanics. Students were generally positive about constructing their own knowledge via these customs and they show reflection about their learning of mathematics. However the radical change from didactical to co-didactical customs required deep changes in the teachers' behaviour and a small number of students found the method inaccessible, although they still felt interested and involved.

This research seems useful, but is very dependent upon choosing appropriate problems for the students to debate and so difficult to do on one's own, without the aid of specialist researchers.

Sierpinska et al (1999) Teaching and Learning Linear Algebra with Cabri

This paper describes a research project in class 3, 'Product-oriented' research. It gives an account of a research programme concerned with the study of a Cabri computer environment designed to help students learn some aspects of linear algebra. The authors' theoretical framework includes the idea of 'multiple representations' in the learning of mathematics. They report that students did not behave exactly as expected and they attempt to explain this. It may be of use to mathematics teachers in that it describes a way in which Cabri might be used and it highlights the successes and shortcomings of their endeavour.

Colgan (2000) MATLAB in first-year engineering mathematics

Whilst this paper does not follow a 'formal' pattern of research (question, methods, results, conclusions), it is typical of much thoughtful curriculum development in class 3 'Product-oriented' work and sharing such experience can be helpful to similar departments

The paper describes a curriculum development in one department in an Australian university, in which the package MATLAB was introduced as an integral part of a first year curriculum in response to a request from the Engineering Faculty. A survey of computer packages and their use elsewhere preceded the decision to use MATLAB, and was followed by the decision to write an appropriate manual as well as course exercises that used the package. The paper reported that feedback was generally good, and the authors believe their experience and suggestions may help others planning similar curriculum developments. However there is no attempt to quantify the effects of the change.

Steen (1994) Twenty questions about Research on Undergraduate Mathematics Education.

This paper is not so much a research paper as a comment on mathematics education research and as such it is belongs to research class 4. It draws out some of the questions that mathematics teachers have about maths education research. The author gives some reassuring answers but leaves many questions for the researcher to deal with. The paper frames the debate that must take place if the teacher and students are to benefit from the fruits of research.

Brown , DeVries, Dubinsky and Thomas (1997) Learning Binary Operations, Groups and Subgroups.

This paper reports on a study of the nature of the understanding of students taking a first course in abstract algebra, in the topics of binary operations, groups and subgroups. The authors presuppose that learning takes place in the APOS framework [Action, Process, Object and Schema] i.e. Students learn by first coming to grips with specific, concrete, examples of a concept, and then proceed to develop (or not develop) deeper knowledge of the concept. Whilst this is 'Product-Oriented' research, it is also to a large extent cognitive and so it straddles classes 1 and 3.

'Discovery' learning methods were used to develop first the idea of the inverse in modulo arithmetic, then identities, associativity and closure to the concepts of group, subgroup and the centre of a group. At the end of the study, the students were assessed, both by formal test and interview. The authors analyse the levels of understanding of the students, and show how these levels of understanding both tie in with the APOS model, and illustrate the process by which the students reached their level of understanding. Whilst the authors explicitly warn against comparing the performance of the students in this study with those taught by standard methods, they do present some evidence that the students involved in the study developed a deeper understanding of the concepts in question than those taught using standard techniques.

This paper is very clearly written and provides results, which agree with the APOS framework of learning. Although it is generally cognitive in nature, it does suggest that discovery based learning is more efficient in developing students' understanding of abstract concepts than traditional methods, which may be of interest to teachers of algebra.

Leron, Hazzan and Zazkis (1995) Learning Group Isomorphism: A crossroads of many concepts.

This paper is concerned with how undergraduate students, in their first abstract algebra course, learn the concept of group isomorphism. The authors discuss two notions of isomorphism. The informal notion of two groups being isomorphic if they are the same except in the names given to the elements and the formal definition involving homomorphism and one-to-one functions.

Various (small) groups of students were both interviewed and examined. Based upon the results of these interviews and examinations the authors stress the role of the informal definition in developing an understanding of the deeper concept of isomorphism, but point out that many students failed to progress from the informal to the formal definition. In particular students often tended to grasp onto those aspects of the concept of isomorphism that they found easier, and neglected those aspects that they found more difficult. For example, whilst most students were comfortable with the fact that isomorphic groups have the same order, many made the false implication that the converse is true. Also many of the students would be quite happy to explain that two groups were isomorphic, using the informal definition, but would be unable to give a suitable homomorphism between them.

Whilst this paper is research in class 2, it is in many ways similar to the previous paper (Brown et al.) which is referenced in the text, in that it is generally of a cognitive nature (class 1). It emphasises the importance of commencing with an informal definition of the concept of group isomorphism. However it is interesting that this strategy does not appear to have been as effective as in Brown et al. perhaps because students found the concept of isomorphism more difficult than the concepts explored in Brown et al.

Meel (1998) Honors Students' Calculus Understandings: Comparing Calculus & Mathematica and Traditional Calculus Students

This paper describes a piece of 'formal' research into the cognitive understanding of mainstream maths concepts (namely limits, differentiation and integration), in two small groups of students at a US university. One group was taught a curriculum based around the Mathematica package, and the other a traditional course. The content of the two curricula was similar but not identical, as the Mathematica based course designedly chose material that would integrate with the package. Evaluation used quantitative and qualitative instruments, and showed slight differences in, largely, predictable areas, namely greater competence with a formal concept for the traditional group, and better problem-solving ability for the Mathematica group, whose exercises had used this form more.

This paper, whilst rigorous, is limited in scale and reveals no surprising conclusions. However, the topic areas are of wide interest, and the methods would be applicable to other studies. It straddles classes 2 and 3: social and epistemological engineering and 'Product-oriented' research.

Orsega and Sorzio (2000) "Deconstructing Rolle": A Teaching Proposal to Foster First Year Undergraduates' Deductive Reasoning.

This is an example of a class 1 investigation into students' powers of deductive reasoning. However it might also be classified as class 2, as the pedagogical context is engineered and evaluated. Instead of a didactic framework a 'mental model theory' was used, in teaching Rolle's theorem. A theorem is seen as a 'logic toy', which is deconstructed to try to help students understand the theorem. This involved the students in undertaking a related set of tasks or experiments: removing each of the three hypotheses of the theorem. The tasks involve examples and counter-examples exemplified graphically. The authors try to analyses students' difficulties. The teaching method was aimed at enabling students to make explicit the properties implicit in the hypothesis of a theorem and to reflect on them. However it appeared that many of the students did not understand some of the underlying concepts e.g. differentiability implying continuity, sufficiency; they were too anchored to graphical representations; and could not connect different fragmented parts of mathematics. The paper highlights some of the problems and educational implications of the research and so would be of interest to teachers considering similar approaches.

The teacher researcher

Mathematics teachers are continually reviewing and updating the delivery of their courses in the light of both internal and external information:

- their own experience of teaching, in particular the relevance of content and effectiveness of delivery, in meeting both module and course aims and objectives;
- student reactions - expressed both informally during a module and more formally in module evaluations;
- assessment results;
- the teaching practice of colleagues;
- accounts of teaching in other institutions;

- research findings reported in professional journals.

Many journal articles provide descriptive accounts of informal teacher research, which can be a valuable source of teaching ideas for other mathematics teachers. Such 'action research' accounts normally include the motivating factors, expressed as research questions; rationales for the ensuing changes; and analysis of the results. These three steps provide the teacher with a more structured method of research and readers are better able to assess the transferability of the approach to their own classroom.

The changes required to update and improve courses may be clearly indicated by the nature of the motivating factor(s) and therefore capable of development and implementation by the teacher without external assistance. However accounts of similar contexts encountered by colleagues can provide new perspectives on such situations and novel approaches to particular problems.

The teacher researcher may well need to consult formal mathematics education research findings when reviews of courses indicate that cognitive aspects of student learning are relevant to course improvement. If appropriate research reports exist these can provide ideas for more radical developments, perhaps based on a non-mathematical discipline.

Collaboration, between mathematics teachers and mathematics educationalists, takes this interdisciplinary approach further and provides opportunities for teachers to influence the focus of educationalists.

Examples of research activities by members of the group.

This section gives summaries of our own experience of and involvement in maths education research, in order to show something of the variety of ways in which mathematics teachers may find mathematics educational research of use. We are not a typical group of maths lecturers (because we *are* all interested in maths education!), but neither are we full-time education researchers. However between us, we have been involved with all four aspects of mathematics educational research mentioned in the introduction. We learn from research studies of specialist researchers (A); some are teacher researchers (B); some have collaborated with specialists (C); and all feel part of the community of mathematics educators (D).

John Appleby has done little formal mathematics education research, but he has published the results of several studies of actual courses and their success (or otherwise!), which fit in the category of 'Product-oriented' research. He has also collaborated in editing the highlights of around twenty years' material from these UMTC conferences (1998a).

An example of incremental change is the change in a first-year matrices course (for engineers), where he almost abandoned formal definitions of 'a matrix', 'a square matrix', 'a diagonal matrix' in favour of 'This is a three-by-two matrix, this is a square matrix, and so is this, ...'. Teaching materials and ideas have also been published (1995a, 1998b).

He has been involved with designing several courses from scratch, mostly lecture-based, but also a modelling course (1995c) and an IT course, attempting to embody his experience from other courses and from what he had read and heard. He used student feedback and final results, with some comments from colleagues.

He was involved with developing a computer-based diagnostic test, which has been extensively used at other institutions. This work started as a TLTP project from 1993-6 but continued after the funding ceased. The research component was to design and validate the underlying expert system and associated question set, and also to design and evaluate the interface. The results of the test and their relationship to subsequent success have also been analysed and published. (e.g. 1995b,1997, 2000). Some of this work (1997,2000) addresses cognitive issues. The test, *DIAGNOSYS*, has been extensively used at other institutions in the UK and abroad.

Judy Ekins has been a teacher researcher, interested in investigating the question of how to make mathematics appealing to both female and male students, since the 1980s, as a member of the Gender and Mathematics Association (GAMMA). This was class 2 research – social and epistemological engineering research. However she did produce some summaries of current research and practice for internal use within her institution (class 4 research).

Within her field of distance education, she is interested in increasing retention and the role of study skills, language skills and the use of different media, to this end (1988, 1989, 1990, 1992, 1994). In the 1990s she became course team chair of a new Open University course *Open mathematics* (MU120), which was aiming to open up mathematics to a wider audience: in particular it aimed to attract more women and adults with lower previous educational qualifications. The aim was to integrate the core skills (later called key skills by Dearing (1997)) of learning how to learn, communication, using a graphics calculator, as well as the use of number. She has been involved with some 'product orientated' (class 3) research relating to this course and its students. She attends relevant conferences, presenting results, which might be of interest to others, as conference papers (1997, 1999, 2000).

Paul Wilson is interested in investigating alternatives to the traditional 'publication' style of lecturing, where the lecture content follows a strict logical sequence commencing with the definitions and theory and working down to the practical examples. His experience has led him to believe that the opposite approach of starting with worked examples and 'working up' to the theory and definitions is more fruitful. He is also interested in determining the broader factors that encourage or discourage students to avail of the educational support available to them (class 2 research).

For example, in a course on differentiation (for students who have covered the mechanics of it before), the aim is that they should understand the concept and not simply do it. Rather than begin with the formal treatment, he would start with a particular example of differentiating a function, then proceed to show how this relates to the rate of change of that function, and from there proceed to develop the more formal theory.

Christine Hill has been involved in a specialist-teacher collaborative research project at University College Chichester, entitled Flexible Learning Approaches in Mathematics in Higher Education (FLAMHE). This contributed to the restructuring of mathematics courses when the College changed from a three term to a two semester academic year. Mathematics education researchers carried out case studies of students' experiences of mathematics courses during their first year as undergraduates and observed and analysed the teaching of particular courses in terms of cognitive and affective issues as well as mathematical understanding (class 1 and 2 research).

This research led to changes in the presentation and content of mathematics course information in the College prospectus, influenced the content of modules offered to students, particularly during their first year, and initiated innovative approaches to course delivery, such as peer-tutoring and use of the College intranet. The range of key/ transferable skills expected by (and from) undergraduates led to an analysis of the relationship between module content, transferable skills and methods of assessment and a more relevant and even distribution of assessment methods across each year of a course (class3 'product-orientated' research).

These changes began to be implemented when courses were semesterized but the process is a continuing one with re-evaluation and further modifications each subsequent year. The whole project represents a collaboration between mathematics teaching and research.

The project was described in more detail in Hill (2001).

Ken Houston has carried out a number of research projects of class 2 and class 3, by himself and in collaboration with others. Much of the research has to do with innovations in teaching and assessment, which have been inspired by reflections on the way of life of practising mathematicians and by the views of employers and others. For example, the Dearing Report (Dearing 1997) recommends that "key skills" should be embedded in courses (Challis and Houston, 2000). Innovative teaching methods were designed to achieve this and then evaluated. Reflection on the ways in which mathematicians work in industry have lead to curricula which include peer tutoring and independent learning methods.

Setting a comprehension test is a way of getting students to read a published article and to work through it. The lecturer sets a written test on the article, setting questions that will tease out the student's understanding of the article. (Houston, 1993, a and b).

Peer tutoring is a widely used method of support for student learning. It can embrace the ideas involved in independent learning. A scheme for using peer tutoring in a mathematics module has been developed over a number of years and refined in the light of evaluation. The scheme now involves students in working in groups, to carry out investigations and learning topics by themselves. They then "teach" their peers through seminar and poster presentations and by writing notes for them to read. (Berry and Houston, 1995, Houston 1995, 1997, 1998, Houston and Lazenbatt, 1996, 1999, Barry et al 1998, Houston et al, 2000).

When the teaching of mathematical modelling had been commonplace for a number of years, a group of teachers turned their attention to developing robust methods for assessing student project work - their presentations and their written reports. This Assessment Research Group (ARG) have developed and tested assessment criteria for these purposes and these are now used regularly by the people involved and by some others. The work was published in four reports and a number of journal and conference papers. (Houston 2001 contains a complete bibliography; see also Haines and Houston 2001; one example is Houston et al 1994.)

Topics for research.

The questions that education researchers ask are not always of direct interest to many maths lecturers, and many questions that maths lecturers would like to ask are either not made public or not expressed at all. The Group identified two areas where we thought research would be useful and we believe is currently inadequate.

What's going on?

It would be good to know what is going on in different areas of teaching. This could be provided by more *surveys of current practice* (research class 4).

If a lecturer perceives a problem with a course or with the curriculum of the department, e.g. lack of student motivation, persistent problems with certain concepts, gaps in background knowledge etc., important questions might be "Is this a phenomenon I can influence?" or "Is there something I'm doing wrong?" If others with similar students and similar syllabuses report similar problems, despite slightly or greatly different teaching methods or assessment, it might suggest that great improvements are unlikely to be achieved even by radical changes in the teaching, except in modest ways. Such a lecturer might still try to improve the students' learning experience, but in a more informed way.

However, if such a lecturer knows that others *are* achieving significantly more, or are not reporting the same problems, it becomes more likely that changes are possible and worthwhile. Even with the fixed constraints of available applicants, resources, and professional curricula, it becomes well-worthwhile investigating the difference in approach and seeking to apply it.

Despite the numerous surveys that have been made of maths departments, there is often a dearth of evidence on what's going on, especially where it's embarrassing (e.g. poor results and scaling of exam marks). Future surveys should concentrate on data that maths lecturers want, which also makes it more likely that they will respond to such requests for information. (There is also little point in surveys that get a very poor response.) It would be more useful to have fewer but more comprehensive surveys (perhaps conducted by telephone rather than post).

What is the most effective and appropriate assessment?

Students have always been driven to a significant extent by the assessment that awaits them. It is probably the case that the amount of assessment has increased, and hence also the driving effect of it. Certainly, the assessment will affect how students study and learn, and will also influence their decision and our advice with respect to their future options and careers. Do we know what we are assessing? How can we distinguish between knowledge, skills and understanding in assessment? How can we assess as we wish without over-burdening both the students and ourselves?

Some research has been carried out (as mentioned above) on the assessment of student projects, but there has been insufficient attention given to the setting of written examination papers. For example, in one school pilot of a sample GCSE question, replacing the word 'rotate' by the word 'turn' doubled the success rate on the question. There are certain to be many such examples at HE level.

Dissemination

As well as producing research results, there needs to be more effort to disseminate what's there. There are several ways this could be done:

> Review papers critically summarising existing research are useful, also books such as 'Current issues in maths education'.
>
> An index of maths education sources (perhaps with a 2-3 line summary of each source, but without the synthesis and judgement associated with a review paper) could be produced, and available on a web-site. A keyword-referenced index, so that papers on a given topic can be found, would be a useful extension to this.
>
> More lecturers should evaluate the developments they *are* making, and to report them. A non-refereed bulletin board might be helpful here with a word-limit on contributions.

Additionally, the length, style and format of publications needs to accommodate the intended reader. Educational jargon is as off-putting to most mathematics lecturers as formal analysis is to most maths students.

It is also important to establish just *why* dissemination is so poor in maths education. Does the fault lie with the writers or with the (potential) readers?

Conclusions.

This report defines mathematics education research and classifies its methodology. It gives examples of different types of research study, which might be useful to mathematics lecturers and gives examples of some uses of mathematics research by the group members. It also makes suggestions for further study.

Mathematics education research has the potential to significantly influence education policy, curriculum design, methods of teaching, and assessment.

Mathematics teachers can and should be involved in this activity, at least through an awareness of relevant publications, and also in being active as teacher researchers and involvement in collaborative research with others who have an expertise in research methods.

References

Alibert, D. (1988) "Towards new customs in the classroom" *For the Learning of Mathematics*, 8(2), pp. 31-35, 43.

Appleby J.C. (1995a) 'Resonance - just a trick?' *Int J Math Educ Sci Technol*, 26, 779-782.

Appleby J.C., Anderson A. et al. (1995b) 'Managing the transition to University education: computer-based diagnostic testing of background knowledge'. *International Congress of Engineering Deans and Industry Leaders, Monash, 1995*. (with others)

Appleby J.C. (1995c) "Making do in Mathematical Modelling". *IMA Conference: The Mathematical Education of Engineers, Loughborough 1994*.

Appleby J.C., Samuels P.C. and Treasure Jones T. (1997) "*DIAGNOSYS* - a knowledge-based diagnostic test of basic mathematical skills" *Computers in Education*, 28, 113-131

Appleby J.C., Burn R.P. and Maher P. (1998a) *Teaching Undergraduate Mathematics*. IC Press.

Appleby J.C. (1998b) "*BUSTLE* - a bus scheduling simulation" *Teaching Statistics* 20 pp 77-80.

Appleby J.C. (2000) "What can we learn from computer-based diagnostic testing?" *Proceedings of TIME 2000 conference* University of Auckland. - to appear

Barry M., Burrell Q., Emery D., Evans C., Gordon N., Houston S.K, Hunter G., Kahn P., Rees J. and Stibbard A.(1998) "The Evaluation of Teaching Innovations" Norcliffe A. (ed) *Proceedings of the 1997 Undergraduate Mathematics Teaching Conference*, Sheffield Hallam University pp39-54.

Berry J.S. and Houston S.K, Students (1995) Using Posters as a Means of Communication and Assessment, Educational Studies in Mathematics 29, pp 21-27.

Brown, A., DeVries, D., Dubinsky, E. and Thomas, K. (1997) "Learning binary operations, groups and subgroups" *Journal of Mathematical Behavior*, 16(3), pp187-239.

Challis N. and Houston S.K, (2000) "Embedding Key Skills in the Mathematics Curriculum" *Higher Education for Capability*, to appear.

Colgan, L. (2000) "MATLAB in engineering mathematics" *International Journal for Mathematics in Education, Science and Technology*, 31(1), pp.15-25.

D'Inverno, R. (1993) "The success of a self-paced course" *International Journal for Mathematics in Education, Science and Technology*, 24(5), pp.727-739.

Dearing, 1997, Report of the National Committee of Inquiry into Higher Education, London: HMSO.

Ekins J.M.(1988) "Study Skills needed for Independent Distance Learning" *Proceedings 5th Annual Conference HKERA*

Ekins J.M.,Tam J.,Tsang E., and Yim K.H.(1989) "Student Feedback on different Media in Distance Education" " *Proceedings 6th Annual Conference HKERA*

Ekins J.M. (1992) "The Development of Study Processes in Distance Learning Students" *Proceedings 1992 AAOU conference* Hong Kong Open Learning Institute

Ekins J.M. (1994) "Economics of Developing or Using Existing Distance Learning Courses in terms of Human Resources" *Economics of Distance Education* Dhanarajan et al. pp199-212. Hong Kong Open Learning Institute

Ekins J.M. (1996) "Embedding core skills in OU Maths courses" *Mathskills Newsletter*, 3, pp 9-12.

Ekins J.M. (1997) "Pedagogical Issues in Opening up Mathematics to a wider audience" *Proceedings of ICDE conference* Hong Kong

Ekins J.M. (1999) "Student re-actions to Learning Mathematics using Different Media" *Proceedings of ICDE conference* Vienna

Ekins J.M. (2000) "Increasing student involvement with their own learning in a level 1 Open University maths course" *Proceedings of the 1999 Undergraduate Mathematics Teaching Conference*, Sheffield Hallam University.

Gavosto E.A., Krantz, S.G. and McCallum, W. (Eds.) (1999) "Contemporary issues in mathematics education", Cambridge: Cambridge University Press

Haines, C.R. and Houston, S.K. (2001) "Assessing Student Project Work", in Holton D. (Ed.), *Teaching and Learning Mathematics at Undergraduate Level*, Kluger, to appear.

Hill (2001) *'The influence of research findings on the re-structuring of mathematics courses at University College Chichester' Proceedings of the 2000 Undergraduate Mathematics Teaching Conference*, Sheffield Hallam University

Houston S.K. (1993a) "Comprehension Tests in Mathematics" *Teaching Mathematics and its Applications*, 1993a, 12, 2, pp 60-73.

Houston S.K. (1993b) "Comprehension Tests in Mathematics : II" *Teaching Mathematics and its Applications* 12, 3, pp 113-120.

Houston S.K, Haines C.R. and Kitchen A.(1994) *Developing Rating Scales for Undergraduate Mathematics Projects* University of Ulster, Coleraine, 35pp. [ISBN 1 85923 0113].

Houston S.K (1995) "Attitudes of Students to Independent Learning" *Humanistic Mathematics Network Journal* 12, pp 41-46.

Houston S.K and Lazenbatt A.(1996) "A Peer Tutoring Scheme to Support Independent Learning and Group Project Work in Mathematics" *Assessment and Evaluation in Higher Education* 21, 3, pp. 251 - 266.

Houston S.K (1997) "Evaluating Rating Scales for the Assessment of Posters" in Houston S.K, W. Blum, I. Huntley and N. T. Neill, eds. *Teaching and Learning Mathematical Modelling: Innovation, Investigation, Applications* Albion Publications, Chichester, pp 135-148.

Houston S.K (1998) "Get Students to do the Teaching!" in Galbraith P., Blum W., Booker G.and Huntley I. (eds) *Mathematical Modelling - Teaching and Assessing in a Technology Rich World*, Horwood Publications, Chichester, pp45-54.

Houston S.K and Lazenbatt A. (1999) "Peer Tutoring in a Modelling Course" *Innovations in Education and Training International* 36 pp 71-79.

Houston S.K, Rodgers P. and Simpson A.(2000) "Teaching Mathematics as a Way of Life", in C. Rust (Ed.), *Improving Student Learning through the Disciplines*, OCSLD, Oxford, 2000, 135-145.

Houston S.K. (2001) "Assessing Undergraduate Mathematics Students, in Holton D. editor, *Teaching and Learning Mathematics at Undergraduate Level*, Kluger, to appear.

Leron, U., Hazzan, O. and Zazkis, R. (1995) "Learning group isomorphism: a crossroads of many concepts", *Educational Studies in Mathematics*, 29, pp. 153-174.

Meel, D.E. (1998) "Honors' students calculus understandings: comparing Calculus and Mathematica and traditional calculus students" in Schoenfeld, A., Kaput, K. and Dubinsky, E. (Eds.) *Research in Collegiate Mathematics III*, Providence, RI: American Mathematical Society, pp. 163-215.

Nardi, E. (1999) "The novice mathematician's inquiry about new concepts: bestowing meaning through ambivalent uses of geometrical metaphors" in Brown, L. (Ed.) *Making Meanings in Mathematics*, pp. 57-68.

Orsega, E.F. and Sorzio, P. (2000) "'Deconstructing Rolle': A teaching proposal to foster first year undergraduates' deductive reasoning" *Teaching Mathematics and its Applications*, 19(2), pp. 69-73.

Schoenfeld, A.H. (1991) "On Pure and Applied Research in Mathematics Education", *Journal of Mathematical Behavior*, 10, pp 263-276.

Sierpinska, A., Trgalová, J., Hillel, J. and Dreyfus, T. (1999) "Teaching and learning linear algebra with Cabri" in Zaslavsky, O. (Ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 119-134.

Steen, L.A. (1994) "Twenty questions about research on undergraduate mathematics education" in Dubinsky, E., Schoenfeld, A. and Kaput, K. (Eds.) *Research in Collegiate Mathematics I*, Providence, RI: American Mathematical Society, pp. 225-229.

Tall, D. and Vinner, S. (1981) "Concept image and concept definition in mathematics with particular reference to limits and continuity", *Educational Studies in Mathematics*, 12, pp. 151-169.

Attracting students to mathematics

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Introduction

"Maths crisis based on teachers shortage [1]." This was a headline in the Guardian newspaper on the second day of the conference. The article opened by stating "Mathematics teaching is in crisis", thus expressing the view of Alison Wolf [2], Professor of Education and Head of Mathematical Sciences Group, University of London who indicates "that the shortage of specialists could generate an innumerate generation". The group is in general agreement with this statement. Members, from their wide and varied experience, feel that there is anecdotal evidence of a shortage of mathematical skills in students. How to attract more students to the study of mathematics and, further, to encourage them into teaching, is therefore of pressing importance.

The study of mathematics develops a wide range of skills, some unique, which can be applied to many other disciplines. Of great importance is that mathematics can be studied for its own intrinsic beauty. A mathematics undergraduate course should, and usually does, allow its students to develop, in a very deep way, an important range of skills: logical and abstract thought, analysis of problems, model-building, together with the presentational skills necessary to convey solutions concisely to other professionals. Communication with industrial placement supervisors indicates that these skills are valued by a whole range of employers, and it should be a source of concern across the whole HE system that the number of applicants for mathematics courses is falling, despite a significant number of candidates sitting A-level mathematics [3]. In 1999 there were 69,945 graded candidates for A-level mathematics of which 83.3% passed with an A - E grade [4]. In 2000 88.9% of the 67,036 graded candidates gained an A - E grade. The corresponding statistics for A-level English are 90,340 graded candidates in 1999 with 92.7% obtaining an A - E grade, 86,428 graded candidates in 2000 with 93.0% passing with an A - E grade. (Please note that figures for 2000 are provisional).

There needs to be a collective effort to attract the right students for undergraduate courses on offer. Alternatively, should we consider changing the content of courses and method of teaching in light of the falling numbers of student applications? Less than 3% of all mathematics graduates enter the teaching profession [5]. Places on PGCE courses are still available this year for mathematicians [6], and this despite short-term enhancements offered by the Government: perhaps mathematics graduates can figure out that a one-off £3000 does virtually nothing to close the salary gap over a whole career!

The brief is concerned with students entering the undergraduate level of study, but exposure to stimulating mathematics at all levels of education, including primary, is important.

Mathematics Departments in Schools

"Only 9% of sixth-formers take A-level maths" quotes Rebecca Smithers in the Guardian [7]. Alison Wolf, in the Guardian Education, goes on to state that "recruitment on to maths PGCE courses had been declining for years" [8].

The teaching of mathematics in secondary schools is carried out largely within a framework of the various Boards' examinations and the prescriptive nature of the National Curriculum. Although good reasons exist for this framework, the most obvious being improvement of standards, the effect on the subject of mathematics may be regarded as negative. The teaching process is driven by a modular assessment regime, resulting in fragmentation of the syllabus. Additionally there is no coherent development of the subject. The beauty of mathematics is then lost, along with the conceptual understanding to all but a few pupils.

A fresh approach to the recruitment and retention of mathematics teachers is urgently required. The enthusiasm of these teachers needs to be maintained with innovative in-service training which should provide lively lessons reducing the prevailing impression that "maths is boring". John Barrow, Professor of Mathematical Sciences at Cambridge and Director of the Millennium Mathematics Project states that "it is important that mathematics is motivated from the world around us in imaginative ways". He suggests that "to teach mechanics start with sport; to get over the ideas of probability, start with the lottery and familiar games of chance" [9].

Primary schools can do much to promote both the love of mathematics and mathematical thinking as distinct from the rote learning of arithmetic. For example, the comprehensive teaching of fractions initiates the transferable skills leading to good algebra techniques, rather than the simple 'cutting up pies' approach.

Careers Services at Secondary Schools and Universities

We feel that many prospective entrants to mathematics courses may not receive sufficient guidance regarding the career choices available to mathematics graduates. The London Mathematical Society (LMS) has produced a pamphlet [10] giving a brief extract from the University Statistical Records, together with a small portfolio of students who have taken mathematics degrees. The pamphlet, however, is singularly coy regarding financial rewards! It is not enough for University Careers Services to be well informed about these opportunities; informed guidance at an earlier age is essential. University departments can do something to remedy this by means of Open Days or involvement in Local Education Authority (LEA) career fairs, but we should also be looking for ways to systematically inform LEA and schools careers officers about current career opportunities for mathematics graduates. It needs to be emphasised that students do not need to have high grades in mathematics A-level to pursue a career in mathematics. Additionally, we as mathematics lecturers must keep ourselves fully informed about current career opportunities. More generally, careers officers need to receive information why employers recruit graduates with mathematical backgrounds. Employers value the skills of logical thought, analysis and problem solving which mathematics courses inculcate. Perhaps the LMS or some other body could produce a similar leaflet setting this out in an enthusiastic manner. Such a pamphlet should perhaps emphasise the 'generic' nature of these courses enabling skills to be transferred to new situations. During an undergraduate degree course a range of mathematical/statistical packages will be used, suited to the particular problems or situations. The flexible skills acquired during their course will make graduates familiar in the use of an appropriate computer based tool to handle a given problem.

All the sciences have a quantitative element, and possession of mathematical skills is an essential co-requisite. It is in our own interest as mathematicians to make common cause with those teaching in related disciplines to encourage schools and syllabus designers to include those elements which support and develop quantitative skills. At the very least this will ensure that the pool from which mathematics students are drawn does not shrink.

The whole of the mathematics community, including schools, further and higher education, LEA, school and university careers services require an integrated approach to careers. Initiatives such as Maths@Work set up by the Institute of Mathematics and its Applications (IMA) [11] as part of the Government Maths Year 2000 Programme [12] will provide school children with valuable information of mathematics in the work environment. Organisations such as LMS [13], the Royal Statistical Society (RSS) [14] and IMA need to continue their work of careers monitoring, support and provision of information. Industry and commerce are essential to this integrated approach and use can be made of the links already existing through university industrial placement units. The skills requirement of employers may necessitate the restructuring of undergraduate mathematics courses.

Subject Content of Mathematics Courses

To address the issue of attracting students to university mathematics, we have to consider the mathematical learning experiences students have at school. In A-level mathematics, some students essentially learn techniques and processes by rote and may not get exposed to more conceptual ideas. This is problematic in two ways. First, some students may choose to pursue mathematics assuming that university mathematics is a continuation of this; they subsequently find the more abstract, conceptual ideas too difficult, or unappealing. Secondly, other students may find little stimulation in learning by rote, and assuming that this 'is mathematics' choose to follow other academic disciplines instead.

School teachers should be encouraged to give an impression of the flavour of mathematical thinking. It is possible to give the layman a feel for concepts without introducing rigour and attempts should be made to do this in the classroom. This possibility is witnessed by the growth in sales of popular mathematics and science books, e.g. The Code Book and Fermat's Last Theorem, two recent best sellers by Simon Singh.

It is up to universities to monitor closely the subject content and syllabus changes of A-level mathematics and restructure undergraduate courses to take account of the differing skills base of students. Students with lower grades at A-level must be encouraged to consider studying mathematics at undergraduate level with courses designed to produce a competent employable graduate. "It is the job of mathematicians and mathematical scientists to provide an accessible and appealing way in to their subjects" [14].

Outreach from higher education to Schools

One way of counteracting the problems concerning subject content and curriculum is to encourage the development of outreach programmes and activities. In this manner, universities and schools can have a closer relationship, and discussions regarding curriculum can be mutually beneficial. Universities can make a useful contribution to schools by highlighting some of the essential skills of a mathematics undergraduate.
With this knowledge, and perhaps via further in-service training, school and university teachers can play an active role in ensuring the smooth progression, in terms of skills requirements, from A-level to undergraduate study.

Various methods can be introduced in order to develop higher order skills, for example the ability to interpret, conjecture and justify, etc. The use of technology in the classroom, with dynamic interactive environments, can encourage students to explore mathematics so providing a vehicle for understanding not only the processes, but also more desirably the underlying concepts. For example, topics such as graphical translations and rotations can be investigated very effectively with the aid of a computer algebra system. Additionally, the use of computers in the classroom can attract students to mathematics who would have otherwise dismissed it as being 'too difficult' or 'boring'[15].

University lecturers can help schools by offering assistance and guidance in the setting of mathematical challenges and Olympiads, further strengthening the liaison between both parties. Mathematics masterclasses aimed at children aged 13 to 14 sponsored by the Royal Institute [16], can provide gifted children with stimulating mathematics. The University of Cambridge through its Millennium Mathematics Project uses new technologies to spread the awareness of mathematics to a wider audience. In particular its web sites NRICH [17] and PLUS [18] provide a wide range of material including puzzles, problems and interviews with people who use mathematics in their job.

Open Days at universities can be a valuable means of 'selling' mathematics as both a useful and beautiful subject, and imaginative presentations can enthuse and inspireprospective undergraduates. In a similar manner, mathematics taster days for potential mathematics undergraduates held at university could also be used to 'sell' mathematics.

Conclusion

It is a matter of urgency that ways are found to attract students to mathematics. We agree with the assessment that the shortfall in appropriately educated mathematics teachers has produced a crisis with damaging implications not only for the viability of some universities mathematics courses (and ultimately, for those who are keen to employ mathematics graduates with their particular blend of skills) but also for the health of a whole range of other related disciplines (and those anxious to recruit their graduates). We have concentrated on a range of ways in which practising professional bodies can help; and although these are small in scale we think it should be openly acknowledged that they all cost time and money - it is time and money we think that individuals, departments, firms and societies should be prepared to spend. We have touched in passing on one key issue, beyond our powers to resolve, that of securing for mathematics teachers remuneration comparable to what their skills can earn 'in the market place'; without significant investment in secondary school mathematics teaching the crisis will continue.

References

[1],[7] The Guardian 5th September 2000, Rosemary Smithers, Education Correspondent

[2],[3] TIKLY,C., WOLF,A. (2000) The Maths We Need Now Bedford Way Papers, London: Institute of Education, University of London.

[4] Student Performance Analysis National percentage figures for A level grades www.bstubbs.co.uk/a-lev.htm

[5],[10]Who Employs Mathematics Graduates? (1994) London Mathematical Society Pamphlet

[6],[8] Guardian Education 5th September 2000, Alison Wolf

[9] BARROW,J.D. (2000) M4aking M4ths Count, RSA Journal, 1/4/2000, pp 4-6

[11] Institute of Mathematics and its Applications www.ima.org.uk

[12] Maths Year 2000 www.mathsyear2000.org

[13] London Mathematical Society www.lms.ac.uk

[14] Royal Statistical Society www.rss.org.uk

[15] MALONE, T.W., LEPPER, M.R. (1987) Making Learning Fun: A Taxonomy of Intrinsic Motivations for Learning, Aptitude, Learning and Instruction: Cognitive and Affective Process Analyses, 1987, pp 223-253

PHILLIPS, R.J., GILLESPIE, J., PEAD, E. (1995) On Designing Screen Images to Generate Mental Images, Exploiting Mental Imagery with Computers in Mathematics Education, NATO ASI Series F, Volume 138, pp225-232

[16] Royal Institution Mathematics Masterclasses www.ri.ac.uk/Schools
[17] NRICH www.nrich.maths.org
[18] PLUS www.pass.maths.org

Use of the internet in teaching mathematics (group A)

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Introduction

There are millions of web pages available on the World Wide Web (WWW). Using the word 'math', or 'maths', in one of the many 'search engines' (such as Alta Vista), brings forth details of many thousands of mathematics web pages. All around the world, mathematicians are more than willing to share their knowledge with others. However, this can present more problems than it solves. For example, what is available? Who checks the veracity of material published on the web? Who decides on its worth? Is the material copyright? Is one likely to be swamped by the sheer volume of what is available? Can the web be used to deliver courses in place of the standard chalk-and-talk delivery? Will this lead to a standardisation in content and delivery?

It is not the intention to provide a review of available teaching and learning material on the web in this group report. Rather, the emphasis is on the likely impact of such material on teaching, learning and assessment and the issues resulting from such impact. For a more in-depth look at mathematics contents of web sites see the other report on mathematics and the Internet elsewhere within this Proceedings. However, the reader is encouraged to visit sites that describe issues relating to the use of web technology in course delivery such as “w4t” [1], COSE (Creation of Study Environments) [2] and WOLF (Wolverhampton Online learning Framework) [3]. The reader can also see details of MLE’s (Managed Learning Environments) and VLE’s (Virtual learning Environments) at such sites as [4], [5] and [6].

What is Available?

Generally, the use of the Internet can be categorised in a number of ways. For example:

- A repository of a lecturer's printed handouts, tutorial questions, etc, quite often still in .txt, .doc, .rtf or .pdf format.
- Pages of tutorial material presented in full web HTML format - sometimes dynamic, interactive web pages, but quite often just an HTML form of notes and handouts.
- Full dynamic web pages quite often with the inclusion of Java applets (small programs that run over the WWW).

- Gateways to other mathematical web sites (e.g. Mathgate [7])
- Publications, Research papers, etc. (e.g. CTI Maths & Stats Newsletter on-line [8])
- Bulletin Boards, Chat Rooms, Telephony, Video Conferencing, etc. for students and tutors to exchange information on their course/module material, administration, etc.
- Major mathematical organisations (e.g. IMA, LMS, RSS, AMS)
- Web sites offering paid tutorial help (or even the completion of assignments!)
- Commercial web sites, advertising mathematical software or perhaps offering software downloads. (E.g. computer-based assessment via Question Mark Computing [9].)

Advantages and Disadvantages of using the WWW

The above list may suggest that there is nothing on the web that can't be done using traditional notes, tutorial software and learning resources centres at a university campus. So what are the **advantages** of using the WWW? Assuming easy access to the WWW at all times, an answer has to be **accessibility, immediacy and quantity**.

- The student no longer has to depend on being present on the right day at the right time 'to pick up the notes'.
- Mislaid notes or solutions to exercises no longer constitute a crisis when they're on the web.
- A workshop tutorial using a particular piece of software needn't be attended if the program is available over the web. (The negative ramifications of this will be discussed later in this paper.)
- Computer animations, illustrative or interactive software can become platform-independent and hence available outside your specific computer laboratory.
- Formative assessments, particularly in "drill" areas can be taken at any time, any place.
- Other material relating to the subject course can be referenced easily, whether it originates in the U.K. or elsewhere. (As an aside, this accessibility also means that it is possible to share your work with others around the world via the Internet, as will be discussed later).
- The Internet is 'immediate'. Using search engines, a student can find and download mathematical material from anywhere around the world at any time of day or night - and the quantity available is immense, as suggested above.
- Delivery of software over the Internet has the important advantage of 'robustness' - students cannot generally interfere with the set-up of software on a web site server and hence the provision of the material is potentially more secure. However, security issues generally remain (see later).
- Up to the time of writing, a search for 'math applets' using Alta Vista has shown a burgeoning number of sites with mathematical applets. Since these are necessarily dynamic, they are more likely to engage the student and, further, give the students sense of 'owning their own learning' . (see [10]).

Since web pages are a dynamic medium, it is possible to ensure that the student can always receive the latest version of any text file or piece of software. Once material is produced in electronic format, it is relatively easy to upgrade. Additional benefits may be that it can then be converted to specific forms for students with special needs, e.g. voice for deaf students; Braille for blind students or large text for visually impaired students. Access to the material 24 hours per day, 7 days per week from other locations than a single dedicated computer lab can also facilitate access for other disabled students. However, there are currently a number of obvious disadvantages in the use of the WWW to consider. For example:

- Despite there being many mathematics web pages, at present there is a problem with producing such pages because of the lack of mathematical symbols available, although there is hope on the horizon in the form of MathML (see for example [11]) - a standard for mathematical 'word processing' on web pages. We would hope that this becomes available soon. In the meantime, one can see web pages with mathematical symbols generated from ASCII characters. Even producing mathematics notes as a text document can cause problems. One of the authors of this report has saved some tutorial worksheets that were converted from Word 97 to 'portable document format' (.pdf), but when downloaded from the web on some Unix systems, none of Word's Equation Editor's equations were displayed!

- Dynamic mathematics tutorial software is readily available on CD-ROM (see Mathwise [12], for example). Up until recently, producing such material over the WWW has proved impossible (except as downloadable files). With the advent of the Java programming language, it has become possible to write programs (applets) that are loaded via the web page to be executed by the web user, [10].

- Since the use of the Internet is rapidly evolving, it is not always possible to keep abreast of all the web sites that can be useful to mathematics lecturers and their students. Consequently it is important to 'bookmark' (save in your list of favourite web sites) important sites that offer guidance on using the Internet in teaching, learning and assessment. In a manner similar to the role of Abstract indices for reviewing mathematical literature, 'gateway' sites are needed. These sites will not only recommend useful mathematical web sites but also offer information on relevant conferences, books, etc. We will discuss further the need for such gateways later in this report.

- Designing and maintaining web sites can be a major issue for staff. In many cases, design and maintenance of web sites falls on the shoulders of the individual lecturer. It would save time and effort if universities or perhaps mathematics communities had some agreed 'web-site template' that allowed easy 'cut-and-paste' development for web page development rather than all lecturers 'doing their own thing'. In some cases, enthusiasm for the benefits of the web can be diminished by the tedium of proper maintenance.

- Although greater access for disabled students has been listed as an advantage, there may be cases when certain types of students are disadvantaged. For example, gender and cultural bias using this technology may need to be considered and addressed.

The advantage of 'distance learning' has been discussed above. However, in distance learning, students talking to students (and lecturers) is still considered important and a potential disadvantage of the WWW is that this communication disappears. There are ways to overcome this using technology. For example, Open University students are given instruction in the use of 'First Class', a package which makes possible the use of threaded e-mail to enable such dialogue. Several distance learning courses make use of the chat-room facility provided by the package WebCT [13]. BLACKBOARD [14] offers similar discussion facilities as well as being an assessment tool. Even within universities, many lecturers are now using e-mail to ensure that all students are made aware of course requirements and commitments and to post course/module notes. This can happen between institutions where the Internet is also being used for telephony (a call to America at local telephone rates, for example) and video conferencing.

Implications of the use of WWW

In many respects the implications of WWW use for teaching mathematics are similar to those faced by lecturers and students using any other distance learning material. The potential advantages of accessibility, immediacy and quantity discussed in section 3, together with the popularity (especially by Government) and mass appeal of the web, have brought these issues into sharper focus.

(a) Teaching

As indicated above, the introduction of web-based resources opens up a wide spectrum of potential teaching arrangements. On the one hand, the lecturer has full control over the presentation of the subject material and the materials produced. Web delivery **complements** traditional lectures with the main advantage being that the materials provided are more accessible as discussed above. Staff remain in control of the whole process on a week by week basis. At the other end of the spectrum stands the complete automated delivery of notes, back-up materials, links and assessment (both summative and formative) over the web, needing no human interaction to run it except for the compiling of essential statistics (read marks) at the end by a well trained operator. In this case, after the module has been set up, the whole process is controlled (in terms of pace of learning) by the student. In many respects, this is how many existing forms of computer-based mathematics teaching software such as CALMAT [15] are used now.

Somewhere in between these two extremes lies a whole gamut of possibilities. A lecturer may have less control over the learning activities when they become more student-centred. There can be different levels of the use of technology, from putting text documents on the web, to generating interactive software, to deepen understanding, and to using web-delivered assessment to manage the students' learning process.

Increasingly, issues concerning the amount and nature of independent learning and other 'graduate skills' are becoming central issues in programme design and quality audits such as Subject Review
(see [16]). Hence the use of the web as an 'independent learning device' (together with an evaluation of its use?) is likely to be an expectation rather than a choice in future programme construction. Web-based activities should not be seen as the only form of learning – student presentations, student group work, etc. should all feature.

It is clear that the extra effort in increasing the access to materials, or indeed introducing access to other materials (elsewhere), is done in the hope that it may facilitate the students' engagement with the course or module. This will very likely happen to some who might find they enjoy it more than before; others certainly will fail to take the opportunity to engage in this way. But, the possibility to make more and different materials available to students may help them to find something that suits them individually in coming to terms with the course or module requirements. What is needed is more examples of **successful** use of web material that can be shared amongst the mathematics community.

It may now be opportune to consider changes to the role of the lecturer. There may well exist very good notes on a topic, or videos of charismatic teachers, all considered to be successful learning materials, so that a traditional oral lecture becomes an unnecessary duplication. In such situations, the local lecturer's role may change to that of managing students' learning either individually or in small groups. This could include monitoring their progress through set assessment tasks, or their attendance. The lecturer's time can be spent diagnosing an individual student's problems or small group problems rather than concentrating on the delivery of subject matter, although there should be no suggestion that staff-student contact time is necessarily reduced. The role of the lecturer is discussed further in the management section that follows.

Staff-student interaction can be on-line, using chat rooms or bulletin boards, to avoid repetition of explanations. This could also encourage student-student interaction, but in an environment which can be moderated by the lecturer. Such schemes often benefit from an element of seeding of the discussions that take place. Using e-mail complements traditional ways of contacting staff and may be more desirable in certain circumstances. But these should not be seen as substitutes for personal contacts between lecturer and students. Technology, like WebCT [13], can help the lecturer by logging assessment scores and averages, how much login-time (and when) for each student, which can be useful at the final assessment.

(b) Student Learning

In an environment where teaching involves asking students to interact with web-based materials, it is likely that students will, in general, have to take more **responsibility for the management of their own learning**. Experience with distance learning material suggests that students should be given regular milestones to achieve (tests, reports, etc.) to help structure their learning. These milestones would allow them to attempt/complete their study at a pace and a time of their own choosing. Students may have to study the same material a number of times before some degree of accomplishment is obtained, thus allowing them to pass on to the next milestone. It could be that different students studying the same material could reach a given milestone at different times. Students should be able to identify their own particular problems and resolve them with a tutor if necessary. In short, the possibility of producing real 'independent learners' exists with this more easily accessible technology. In early years, student access to the system and/or student attempts at formative assessments may need to be monitored to help the lecturer identify problems early on.

As indicated, 'virtual laboratories' offer the possibility of student 'learning by doing' or 'active learning' rather than the passive learning that can happen in a purely lecture-based course. Indeed, ultimately the student may have the freedom to make informed decisions that control the amount of formal attendance (if any at all) at an H.E. institution he/she requires to complete a course (including the assessment process). Formal meetings with personal tutors may be the only formal attendance requirement.

It is considered unlikely that this extreme, of almost complete remoteness of students from university and lecturing staff, will prevail. Nor indeed is it considered desirable as much anecdotal evidence suggests that student's learning benefits from social interaction with each other as well as interaction with teachers in tutorial sessions. Any move towards increased independent/distance learning should be monitored carefully and is considered more likely to be seen in the later years of an undergraduate course (in a manner similar to the demands of final year project work). The potential step-change in student learning habits from school/college to first year at university could be harmful to students. Mathematics education research could be useful in identifying student- learning issues in such a learning environment.

(c) Assessment

In theory, the remote access possibilities of the web could lead to some or all assessment taking place at anytime or in any place, if software based assessment is used. Formal written examinations at a set time of the year could be replaced wholly or in part by a computerised assessment, which is made accessible to a given student at a given time with the agreement of assessor and student. Different students on the same course could be similarly assessed at different times, each student having a slightly different computer-generated assessment. Web-based software such as Question Mark PERCEPTION [9] allow this in some form and one of the authors of this report has had experience of so using PERCEPTION to assess a class of 150+ engineering mathematics students rather than using a formal written examination paper. In the same way, referrals could be attempted a number of times (although this idea of considering assessment like a 'driving test' may not be considered desirable). Disabled students could benefit from easier access to assessment material.

Video conferencing via the web would allow the possibility of remote vivas taking place and the recording of such vivas if required for future confirmation of correct examination protocols.

Computer-based assessment has the potential advantage of automatic and immediate marking and feedback to an individual student. Also, it may be possible (desirable?) to share questions and/or examination papers in common modules between Universities so as to reduce preparation time and maybe help to ensure comparable standards between universities. It is not considered likely that the above scenario, even if desired, is attainable in the near future. Reasons for such a statement include:

- The input of mathematical symbols and expressions on the web is not yet easy or standard.
- Setting computer-based questions, even in multiple-choice form, is not straightforward (if done properly) and does not cover all aspects of assessment. Hence there is still a strong need for written examinations involving such things as proofs, for example.
- Plagiarism, an issue unresolved now, is potentially more of a problem with remote access assessment, even if video conferencing is employed.
- The technology needs to be robust and secure to ensure not only that students can access the actual assessment but that a database containing student marks and other assessment records is maintained with access available only to appropriate people. Assessors can become a hostage to technology (see management section below).
- A heavier reliance on computer-based assessment could disadvantage some students who lack I.T. skills and who may not be able to finance their own individual web access.
- The whole issue of degree classification, number of referral opportunities, number of final year examinations sat at a time, etc. would need to be reconsidered at both local and national level.

New initiatives like AIM [17] do try to extend the functionality of computer based assessment for mathematics by using the power of computer algebra packages.
A number of issues regarding attempts at more localised computer-based assessment (not necessarily web-based), have arisen and been reviewed by Race et. al. [18].

(d) Management

As described above, there is a continuum of possible models of the use of the web, each with its own "management" issues, from the relatively simple one in which only the notes for the course are on the web and are used to augment the lectures, through to the 'all-singing all-dancing' module in which all the material is on the web, making full use of the audio and video capabilities, and with links to other web-sites for individual parts of some (or all) of the topics. Each model has its own validity, related to the purpose of the module and the intentions of the lecturer, and may thus represent "good practice".

There can be a problem if the student becomes too involved in the whole "web-process". As suggested above, it is advisable to set frequent milestones so that students have a framework in which to schedule their work. The need to plan the format of the course will increase because of the greater variety of possibilities and pathways. Obviously, the course/module guide to students at the start of the course/module should outline the format in detail.

The role of the lecturer may change from the "conventional" stand-in-front-of-the student-person to that of a "learning manager", i.e. someone who suggests pathways for learning to the students and who can act as a safety-net for those who experience problems. The lecturer may choose the pace with which the students are exposed to different ideas from other web sites and how these relate to the "home"-course. Providing a selection of vetted web links may help students daunted by the seemingly endless supply of materials and guard to some extent against exposure to inferior quality materials.

The increased variety of presentation offered by the use of the Internet may attract students with increasing diverse enrolment patterns; there may be more students wishing to enrol for part-time degrees or even for only individual modules. Lecturers will have to be able to organise the management of the module to cope with this diversity and the central administration of the university will also have to have in place systems for keeping track of the progress of individual students.

This control will vary (should vary?) for students in (say) first year courses/modules where recommendations to access the web may be very proscribed (e.g. visit site A and comment on the approach given there) and for students in later years where, one would hope, they have developed a level of independence and control over their own learning and its pace. It would be advisable, especially at the first year level, for the course handbook to be very explicit about what the students must do, what they should probably not do and what milestones have been put in place. The Open University course handbook is an example of this kind of material.

Accessibility using the Internet means not only that materials can easily be shared but also that the development costs of those materials can be shared. (Those wishing to keep their material within their own institutions can place the material on an institution-wide Intranet.) Whilst it is clear that there may be an overall increase in the workload for lecturers if they work in isolation, there is a potential advantage in a model in which lecturers from different institutions co-operate over the preparation and delivery of a web-based course. This co-operation could include the preparation and delivery of web-based assessment and of the resulting feedback to the students. The organisation of such courses is, itself, made simpler by the web where lecturers at – different institutions can view and then comment upon draft web-pages prepared by their colleagues; the exchange of information has been made much easier by the advent of the Internet. In fact (and this may not be considered to be an advantage!) the process of quality control (e.g. through the QAA) may become easier if the External Examiners can view the courses through the web.

With the development of video conferencing, the idea of sharing courses across universities becomes viable. In particular, this could be a way to broaden the provision in a subject area independent from the actual staff expertise in a given location. Although progress is being made in providing the infrastructure to enable this, the current financing arrangements and the competitive nature of HE Institutions may impede progress on the implementations of these ideas.

Individual lecturers will want to think very carefully before embarking on a project to set up a web-based course. They will, of course, wish to consider the points raised above and in other sections of this report and come to a judgement of the viability of the project in terms of the proposed outcome from the project compared with the effort that would be needed for its creation. They will also need to bear in mind the institutional support that they will have available to them, whether or not the appropriate resources (software, hardware and technical support) are readily available.

Consideration should also be given to any issues of copyright and intellectual property rights and to whom they belong. Once you have published the material on the Internet it now becomes available to a much wider audience. This, of course, can cause additional problems such as the author being now legally responsible for the material. This may need the lecturers to seek advice (possibly legal) and, conceivably, for a contract for the preparation of the module to be entered into with the university. This problem is more likely to arise when it is thought that the material might be attractive to an audience outside the institution concerned.

(e) Tools

The main problem associated with the development of material on the Internet has been the necessity to learn and produce HTML. Over the past few years there has been a variety of software tools which have made it readily possible for users to transfer their static lecture notes very easily onto the web via Word, Latex etc. The software currently available appears to be in its infancy but developments are occurring at a significant pace. One of the major factors influencing the development of Web based material is time and most current Web authoring packages require a considerable amount of investment before one can become proficient in the development of Web based material. There is significant scope for the development of a standard Web authoring tool which is both easy to use and has the capability to generate mathematical equations and allows the user to experiment with different standard Web page designs through the use of a development kit. As noted earlier, there are still problems in the production of mathematical equations and the need for platform-independent software which will allow the user to view the Web pages.

While most producers of academic web content will be familiar with Word and Latex for document presentation on the web and some will be familiar with HTML, few will be familiar with programming dynamic mathematical software for web. Here the would-be programmer is confronted with having to learn a programming language such as Java. Those already familiar with C or C++ will not find this too onerous whilst novices will want to hire the services of a professional programmer – so incurring possible cost. It is possible to buy software translators that transcribe programs written in Visual Basic, for example, for web delivery. However, Java is the most widely adopted approach since the latest versions of web browsers such as Internet Explorer and Netscape are capable of running Java software through the browsers themselves.

At present there is little monitoring or even a consistent strategy for producing or, for that matter, locating mathematical topics on the Internet. It would be a considerable improvement if a repository of Internet sites was produced and information on them consistently stored via key words and Meta tags which were recognised standards in the Mathematical community. This would then allow the development of new bespoke mathematical search engines, such as MathGate [7], to locate specific mathematical web sites. The database would have hyperlinks to the Web sites and would contain a standard front page indicating the content and use of the material contained within the site. A peer review process would also form an essential part of feedback on the material and the basis for further development.

(f) Resources

A move to Internet delivery of course material would have a significant financial impact on an academic institution as the majority of staff would have to become proficient in the use of this new technology, even if tools were developed to make web page design easier. 'e-learning tools' such as Lotus Learning Space [19] and the e-learning products promised soon by Microsoft may help to produce coordinated , Institutional-wide, web-based learning environments at a reasonable time cost to the individual academic staff member.

There is also a need for 'standard' software (computer algebra software such as Maple, Derive, Mathematica etc., IT software such as Word, Excel, etc. ...) to link up seamlessly with Internet browsers so that a student remote from a university campus can use all the software tools required with a minimum of fuss and delay.. Financial strategies for web based materials need to be implemented at institutional/governmental level.

The Future

The implementation of the National Grid for Learning over the next few years will raise the level of expectations on web-based materials with which students will arrive at university. So Higher Education must embrace the opportunities the Internet provides.

As described, the implications for change in teaching and learning are enormous. However, care must be exercised for at least two reasons:

- technology changes so rapidly that any heavy investment in a current technology or methodology may soon be superseded, e.g. the impending introduction of MathML
- there is as yet limited evidence of the benefits for the students of web-based learning and assessment.

We look to organisations such as LTSN (MSOR), together with other mathematical societies both national and international, to initiate research into the effectiveness of web-based learning, to disseminate the result of that research, and, through the LTSN staff development program, to train staff in the emerging technologies. We also look to them to assist in the evaluation of available materials, to encourage peer review and the establishment of common standards.

We take it as a given that the control over the teaching and learning processes using web-based materials remains with academic staff

Summary of Issues

Using the Internet could, for example, make it possible

- to increase the interaction with students on tutorials
- to increase the amount of one-to-one teaching
- to guarantee experts are lecturing on advanced topics
- to allow a more careful management of student activities where appropriate

On the other hand, the introduction of the Internet

- may tempt managers to discard human resources, i.e. academic staff
- will most likely lead to a change in the type of activities that need to be done, and, consequently a redeployment of staff and other resources
- requires very high set-up costs for the initial provision of courses

References

[1] http://www-maths.cudenver.edu/w4t,
[2] http://www.longman.net/COSE
[3] http://www.wlv.ac.uk/wolf/
[4] http://www.jisc.ac.uk/pub00/c07_00.html
[5] http://www.focus.co.uk/
[6] http://www.ctt.bc.ca/landonline/sbsall.html#tabletop
[7] http://www.mathgate.ac.uk/
[8] http://ltsn.mathstore.ac.uk/newsletter/default.htm
[9] http://www.qmark.com/products/index.html

[10] Edwards, P., 2001, Mathinsite: Java Maths on the Web, MSOR Connections, Vol 1 no 1.
[11] http://www.w3.org/Math/
[12] http://www.nag.co.uk/projects/mathwise.html
[13] http://www.webct.com/webct
[14] http://www.blackboard.com/
[15] http://www.maths.gcal.ac.uk/calmat/
[16] http://www.qaa.ac.uk/
[17] http://calculus.rug.ac.uk:8080/
[18] Race P., Brown S. and Bull J (eds), 1999, Computer Assisted Assessment in Higher Education; Kogan-Page, London.
[19] http://www.lotus.com/home.nsf/welcome/learnspace

The use of the internet in teaching mathematics
(group B)

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Introduction

The main purpose of this paper is to provide teachers of mathematics in higher education with a guide to a selection of web-based resources which it is hoped will be useful in their teaching. Short reviews of a number of web sites are provided. It is not claimed that the collection of sites reviewed is exhaustive, or indeed that those reviewed are the best available. However, it is hoped that the present work will at least give an impression of the wealth of material that is available on the web and stimulate the reader to investigate further. It should be noted at the outset that many more sites were visited by members of the working group than have been included here; we have included those sites which we consider have something to offer the practising lecturer.

We begin with a short discussion of some of the issues surrounding the use of web-based resources. Firstly, it is noted that there is a great deal of software now available to support the teaching of mathematics. The present working group was not concerned with these, but wished to isolate those issues that are particularly important to *internet*-based resources as opposed to more general I.T. resources.

It was noted that examples of good internet resources abound, with excellent web sites existing on, for example, the history of mathematics, the properties of the Fibonacci numbers, fractals, general encyclopaedias of mathematics and interactive tutorial sites. Certainly there appears to be no shortage of relevant and interesting sites. Commercial environments are now being developed, which support general use of the internet, providing teaching resources, such as files and course outlines, discussion areas, and even on-line testing: see **http://www.blackboard.com/** for example. However, these are often too general in scope, and lack the kind of support specifically needed for mathematics.

There are a number of ways in which such resources might be incorporated into courses in mathematics. At the most modest level, a broadly traditional course might be enhanced by references to suitable web sites given throughout the teaching offered to students. Such an approach would add an extra dimension to the study of the syllabus by encouraging students to explore differing approaches to a given topic, without detracting from the merits of traditional teaching methods.

A more ambitious aim would be to use internet resources to totally, or at least partially, replace conventional methods of providing tutorials and student assessment. Pursuance of this aim would have some substantial ramifications. In particular, it would be expected that increased use of internet resources would lead to a corresponding decrease in the amount of time spent attending conventional tutorials. This would have some advantages, in particular it would give students greater flexibility of times of access to resources and allow each student to proceed at his or her most comfortable pace subject to meeting deadlines set by the tutor responsible for the course. On the other hand, it would also present the need to closely monitor students to ensure that no student was either becoming discouraged or failing to maintain their commitment to their course for want of sufficient interaction with tutors. It is certainly not the wish of the group to suggest that internet resources - particularly for on-campus students - are a total replacement for the physical presence of a teacher of mathematics!

Effective use of web-based resources requires the provision of hardware and software which reach a certain minimum specification. The speed of development of both hardware and software mitigates against giving definitive specifications but the group are agreed that provision should satisfy the basic criteria that it must be sufficiently fast and reliable to encourage rather than discourage student use of the resource. There are several issues which need to be addressed by those responsible for resource provision.

Firstly, staff and students need to have a sufficient number of PC's available whose specification enables them to run a suitable web browser efficiently. Although most institutions now meet this requirement, it cannot always be taken for granted. There is also a need for suitable embedded equation editors, and email facilities that allow the interactive communication of mathematical symbols.

Secondly, the provision of electronic whiteboard facilities, and even graphics tablets, can assist in the remote interaction required to communicate mathematical ideas between teachers and learners and within groups of learners.

Thirdly, there is a need to cultivate in students adequate search skills to enable them to locate the web resources required without the onset of the "lost in cyberspace" syndrome resulting in the loss of so much valuable time.

Fourthly, the web offers both teachers and students the chance to make materials available without any of the usual constraints of printed media, so that materials used in the learning process may never have been scrutinised before being made available. Students often assume that materials they read are true, when in fact they may be wrong, and this can lead to many lost hours trying to understand an erroneous statement. Students need to allow for this by developing their critical faculties together with a healthy academic scepticism particularly for information presented to them which is unsupported by solid evidence.

Fifthly, the web does provide materials that are created and supported elsewhere, however this can lead to problems. For example, the use of unusual (and not locally available) fonts in documents, the demands made by animations, video clips or different paper sizes, can mean that actually running or printing learning material may be a time consuming and frustrating experience for students.

Finally, there is a need for reliability in web-based resources. It is hoped that important web sites will be backed up by the provision of mirror sites that may be accessed if the original site fails or is removed - the web is a dynamic series of links that work today but may not be available, or even exist, tomorrow. There is also a substantial need for technicians and technical support for staff and students using web-based resources to ensure that all users are able to make the most of what is available. Addresses specified by staff for student use should exist even in cached form at least for the duration of the course being studied.

A number of mechanisms already exist for identifying good web sites and promulgating them across the community. In particular, organisations such as the Learning and Teaching Support Network for Mathematics and the Institute for Learning and Teaching could act as centres to be accessed by staff wishing to view potentially useful sites; perhaps UMTC could itself provide a place for materials to be gathered and knowledge shared, although it would still need enough technical support to make this available and keep it maintained. The group suggest that the mathematics education community will need to accept responsibility for the creation and maintenance of such a resource, since no individual will have the time to devote to such a time consuming operation. Currently, there exists a number of sites giving extensive lists of links to useful web pages. It may also be useful if mathematics departments produce current bookmark lists of useful sites for their own students, such as those reviewed in the present work.

Reviews of Web Sites

The following sites represent some of the better sites visited by group members during the course of UMTC2000. The sites have been grouped according to their mathematical content. The group feel that it is worth making the general point that limiting searches to **.ac.uk** reduces the amount of learning material found substantially. It also seems to produce few actual on-line learning materials, apart from lecture notes on the web.

Group 1 - General Interest

URL: http://www.mcs.surrey.ac.uk/personal/R.Knott/Fibonacci/fib.html
Title: Fibonacci sequences
Audience: School and College

Content: A plethora of interactive pages, for example: Fibonacci Numbers and Nature; the Golden Section in Nature; Fibonacci Puzzle Pages; the Mathematical Magic of the Fibonacci Numbers.
Strengths: An excellent, extensive site with plenty of interesting information and interactive slots that have to be visited to be appreciated.
Weaknesses: N/A.

URL: http://archives.math.utk.edu/topics/
Title: Topics in Mathematics
Audience: All levels
Content: Listings of link to WWW resources in mathematics, arranged by topic, with indications of the level of mathematical background assumed and of features such as interactivity, animations, etc. for each site.
Strengths: An excellent, comprehensive site
Weaknesses: N/A

URL: http://www.dcs.warwick.ac.uk/bshm/resources.html
Title: History of Mathematics
Audience: All levels
Content: Links to Web Sites on the History of Mathematics
Strengths: Comprehensive and still being developed
Weaknesses: N/A

URL: http://www-history.mcs.st-and.ac.uk/history/
Title: History of Mathematics
Audience: All levels
Content: The MacTutor History of Mathematics archive.
Strengths: A most extensive collection of biographies.
Weaknesses: N/A

URL: http://www.geom.umn.edu/
Title: The Geometry Centre
Audience: All levels, including teachers
Content: Gallery of Interactive Geometry.
Strengths: A number of graphical images available but more novelty than pedagogy.
Weaknesses: Non-interactive

URL: http://www.pbs.org/wqed/lifebythenumbers/standard/index.html
Title: Life by the Numbers
Audience: All levels
Content: Patterns of nature
Strengths: General interest
Weaknesses: Limited scope

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URL: http://archives.math.utk.edu/popmath.html
Title: POPMathematics

Audience: School and College
Content: General interest
Strengths: Interactive
Weaknesses: Requires a download of software

URL: http://dir.yahoo.com/Science/Mathematics/
Title: Index of Mathematical Topics
Audience: All levels
Content: Directory of mathematics resources
Strengths: Comprehensive
Weaknesses: N/A

URL:http://www.mathsource.com/Content/Applications/Engineering/Electrical/0204-
Title: Library of Mathematical Materials
Audience: Second and third year
Content: Engineering mathematics
Strengths: An excellent interactive source site for various aspects of undergraduate mathematics.
Weaknesses: Limited scope

URL: http://forum.swarthmore.edu/library/
Title: Internet Mathematics Library
Audience: All levels
Content: Mathematics resource listing
Strengths: Wide ranging
Weaknesses: N/A

URL: http://www.ex.ac.uk/cimt/
Title: Centre for Innovation in Mathematics Teaching
Audience: Schools and Colleges
Content: The Centre, established in 1986, is a focus for research and curriculum development in Mathematics teaching and learning, with the aim of unifying and enhancing mathematical progress in schools and colleges.
Strengths: A good example of an informative site.
Weaknesses: Non-interactive

URL: http://www.ams.org/mathweb/
Title: Math on the Web
Audience: Year one courses
Content: Maths on the web.
Strengths: A First Stop for finding mathematical sites on the web
Weaknesses: Non-interactive

URL: http://spanky.triumf.ca/
Title: The Spanky Fractal Database
Audience: Year two and higher courses

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Content: Fractal Geometry
Strengths: Good interactive site
Weaknesses: Mainly graphical with little mathematical content.

URL: http://www.cs.ubc.ca/nest/imager/contributions/scharein/KnotPlot.html
Title: The KnotPlot Site
Audience: Specialist courses on Knot Theory
Content: Animations of knots amongst other items
Strengths: Good animations and extensive
Weaknesses: Mainly graphical with little mathematical content.

URL: http://www.shu.edu/html/teaching/math/reals/reals.html
Title: Interactive Real Analysis
Audience: First and second year courses
Content: Interactive Real Analysis is an online, interactive textbook for Real Analysis or Advanced Calculus in one real variable. It deals with sets, sequences, series, continuity, Lebesgue, topology, and more.
Strengths: Comprehensive coverage
Weaknesses: Non-interactive

URL: http://www.cut-the-knot.com/front.html
Title: Interactive Mathematics Miscellany and Puzzles
Audience: School and College
Content: Interactive mathematics in a variety of topics
Strengths: Lots of interest and a variety of interactive topics
Weaknesses: Not mathematical in content, more illustrative.

URL: http://www.teach.virginia.edu/teacherlink/math/links/interactive.html
Title: Interactive Projects
Audience: All years
Content: Compendium of web sites offering interactive mathematics.
Strengths: Large list of web sites.
Weaknesses: N/A

URL: http://SunSITE.UBC.CA/LivingMathematics/V001N01/UBCExamples/
Title: Living Mathematics
Audience: All levels
Content: Lots of interactive mathematics
Strengths: Interesting site
Weaknesses: No consistent theme - a collection of isolated topics.

A maths search engine: http://www.maths.usyd.edu.au:8000/MathSearch.html
The original CTI Mathematics site: http://www.bham.ac.uk/ctimath/
The LTSN Maths, Stats & OR Network http://www.bham.ac.uk/msor/
Information on technology in teaching: http://www.hull.ac.uk/mathskills/

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For some articles dealing with the more general issues of computer-based learning (how to use it, how to measure its effectiveness) see also http://www.hull.ac.uk/mathskills/umtc/

Group 2 - Linear Algebra

URL: http://forum.swarthmore.edu/linear/linear.html
Title: Math Forum - Linear Algebra
Audience: More suitable for teaching staff (American biased)
Level: Assorted
Content: List of links (with reviews) to hundreds of other sites. The site claims to be "The best Internet resources for linear algebra: classroom materials, software, Internet projects, and public forums for discussion." Doesn't actually provide content, but gives a "yahoo" type list of sites with reviews.
Strengths: Very good selection - covers a wide range of topics.
Weaknesses: Possibly too much information - hard to find specific examples. Many examples are basically lecture notes, together with programs for specific packages, and so are of limited use.

URL: http://www.sisweb.com/math/tables.htm#top
Title: Dave's Math Tables
Audience: lecturers and students
Level: First year students
Content: some mathematical theory, and some tools for teaching over the internet
Strengths: tools - the whiteboard for interactive tutorials. Message board for students to use.
Weaknesses: very limited mathematical content.

URL: http://www.cougati.ab.ca/
Title: Felynx Cougati Topics in Mathematics
Audience: Undergraduates
Level: First year students
Content: Online teaching materials covering Linear Algebra - vectors, scalar products, etc.
Strengths: Quite a lot of material, quite good theory, good interactive/multimedia use
Weaknesses: Gets a bit awkward with lots of open windows, and quite slow.

URL: http://www.math.tamu.edu/~stecher/Linear-Algebra/
Title: Topics in Linear Algebra
Audience: Undergraduates
Level: First year students
Content: encyclopaedia of basic concepts, examples, and techniques.
Strengths: reasonable structure, some useful examples
Weaknesses: really just lecture notes. Also, the link to class materials (potentially the most useful part), doesn't work!

URL: http://www.horizonweb.com/js/equat/line_alg.html
Title: Java Script Linear Algebra
Audience: Undergraduates
Level: First year students
Content: Some Java programs to do basic linear algebra operations such as finding determinants and performing matrix operations, along with explanations of the mathematics.
Strengths: Interactive
Weaknesses: Awkward interface, very limited in what operations you can do.

URL: ftp://thales.cs.umd.edu/pub/Jampack/Jampack/AboutJampack.html
Title: JAMPACK: A JAVA PACKAGE FOR MATRIX COMPUTATIONS
Audience: Lecturers
Level: N/A
Content: Collection of co-operating classes designed to perform matrix computations in Java applications.
Strengths: Useful resource that could be incorporated into teaching materials
Weaknesses: Needs downloading and setting up locally.

URL: http://www.7stones.com/Homepage/Publisher/linAlgMenu.html
Title: Visual Linear Algebra
Audience: General
Level: First year students
Content: interactive graphical demonstration of effects of linear transformations.
Strengths: Shows how altering transformation matrices affects the image of the maps. Immediate results of changes.
Weaknesses: Little mathematical explanation.

URL: http://www.public.asu.edu/%7Esergei/linalg/LinAlg.html
Title: Linear Algebra Home Page
Audience: Staff and students
Level: First year students
Content: Some materials to support the teaching of Linear Algebra at Arizona State University, along with some links to other sites.
Strengths: Good example of the kind of layout for such a site
Weaknesses: Very specific (but appropriate for this case!)

Group 3 - Dynamical Systems

URL: http://www.cs.brown.edu/research/ai/dynamics/tutorial/home.html
Title: Learning Dynamical Systems: a Tutorial
Audience: Staff and students
Level: Not specified, but suitable forhigher undergraduate students

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Content: Based on lecture course by William Shaw at Brown University. Syllabus given as topics for each of 30 lectures; on-line lecture notes provided for most of these lectures (but some unavailable or under construction), plus reading list (including some on-line articles), and exercises involving use of Mathematica notebooks.
Strengths: thorough coverage of subject at a high level (final year / postgraduate); comprehensive list of references to further reading.
Weaknesses: incomplete; requires availability of Mathematica; not interactive.

URL: http://www.math.okstate.edu/mathdept/dynamics/lecnotes/lecnotes.html
Title: Dynamical Systems and Fractals Lecture Notes
Audience: Students
Level: Appropriate to final-year undergraduate
Content: Lecture notes for one-semester course, with problems; by David J. Wright, University of Leeds. Uses Maple and Fractint software (the latter freely available on the Web) for examples. Long reading list provided.
Strengths: Well-organised set of lecture notes, with plenty of references (but almost entirely to print materials); good graphics, both within lecture notes and by use of Fractint.
Weaknesses: Not interactive, except when using Maple and Fractint.

URL: http://www.ifs.tuwien.ac.at/~aschatt/info/ca/ca.html#Introduction
Title: Cellular automata
Audience: Staff and students
Level: Probably aimed at postgraduates.
Content: Tutorial on cellular automata by Alexander Schatter, University of Vienna.
Lecture notes giving comprehensive introduction to theory and examples of cellular automata.
Strengths: Well organised tutorial to a fairly advanced topic in dynamical systems.
Weaknesses: Not interactive; use of English sometimes of dubious quality.

URL: http://www.cnd.mcgill.ca/computing/doc/xpptut/start.html#toc
Title: XPP Tutorial
Audience: Staff and students
Level: Higher undergraduate students
Content: Tutorial and examples using XPP (a system for solving differential and integral equations using phase-plane analysis) to study dynamical systems.
Strengths: XPP gives good graphical presentations of behaviour of dynamical systems.
Weaknesses: Requires familiarisation with XPP before starting work on dynamical systems; not suitable for basic studies of the theory.

URL: http://trixie.eecs.berkeley.edu/~chaiwah/introduction.html
Title: Introduction to Non-linear and Chaotic Phenomena
Audience: Not Specified
Level: Final year undergraduate students
Content: Brief introduction to non-linear dynamical systems, by Chai Wah Wu, University of Berkeley.

Strengths: Clearly presented with some good graphics.
Weaknesses: Very brief introduction: only 4 pages of notes; some of the graphics take a long time to download (some failed); not interactive.

URL: http://alamos.math.arizona.edu/~rychlik/557-dir/index.html
Title: Introduction to Dynamical Systems and Chaos
Audience: Not Specified
Level: Final year undergraduate students
Summary of main points from each lecture on a 36-lecture course at University of Arizona.
Strengths: References to earlier equations are hyperlinks.
Weaknesses: Only one or two lectures actually have a set of notes from which one can learn; the remainder just have a series of bullet-points, or nothing at all.

URL: http://mtl.math.uiuc.edu/modules/discrete/index.htm
Title: Discrete Dynamical Systems for Mathematics Teachers
Audience: In-service mathematics teachers.
Level: Not specified but see below.
Content: Distance-learning module from University of Illinois, very thorough tutorials on elementary theory of dynamical systems (appropriate to first year of a mathematics degree), with examples and exercises to be done on a TI-82 calculator.
Strengths: Complete module: no other instructional material required; clear explanations of theory, with good integration of examples and exercises.
Weaknesses: Requires particular calculator.

URL: http://www.ldeo.columbia.edu/~mspieg/Complexity/Problems/Problems.html
Title: An Introduction to Dynamical Systems and Chaos
Audience: Not specified
Level: Not specified
Content: Tutorials covering the fundamentals of dynamical systems and chaos in an informal manner, with examples involving use of the STELLA package, by Marc Spiegelman, Columbia University.
Strengths: Easily intelligible explanations of the basic concepts.
Weaknesses: Does not go very deeply into the theory; not interactive; not clear who it is aimed at.

Group 4 - General Calculus

URL: http://www.mathacademy.com/platonic_realms/encyclop/articles/dif_rule.html
Title: Rules of Differentiation
Audience: Anyone wishing to look up the rules of differentiation
Level: First year students
Content: Rules of Differentiation and many other rules of the calculus
Strengths: None
Weaknesses: List of the common results, no derivations, no examples.

URL: http://ccwf.cc.utexas.edu/~egumtow/calculus/
Title: Help with calculus for idiots
Audience: College students, especially those majoring in subjects other than mathematics.
Level: First year students
Content: A "read-through" exposition only. Written from the point of view of a student, explaining many topics in calculus in a way that fellow students will understand.
Strengths: None in particular
Weaknesses: No example sheets for students to try, no animations, no student support to speak of.

URL: http://web.mit.edu/wwmath/index.html
Title: Introduction to the calculus (MIT)
Audience: Undergraduate students at the time of writing
Level: First year students
Content: Starts with the basics and develops the topic as a whole. Seems to concentrate on the exposition but does give some examples. Worth a look but the site looked at was by no means complete.
Strengths: Appears to be fairly comprehensive in its long term aim.
Weaknesses: Very few examples for students to try and those that were looked at were not interactive in any way. The package gave the problems and the answers but, for example, no credit rating or second chances. The examples did not contain variable parameters or feedback mechanisms

URL: http://www.hofstra.edu/~matscw/RealWorld/Calcsummary4.html
Title: Applications of the derivative
Audience: Undergraduate students
Level: First year students
Content: Mainly elementary applications of the derivative but does include some "step-by-step" approaches to problem solving which students may find useful and informative.
Strengths: Diagrams and a step by step approach, detailed worked examples which should prove useful to students.
Weaknesses: As is often the case, there is insufficient material for students to practice. Some attempt is made at giving feedback but this is a rather weak area.

URL: http://www.mathforum.com/
Title: Mathematics Forum
Audience: Undergraduate students
Level: First and second year students
Content: Very variable, from, for example pre-calculus algebra to multivariable calculus - a compendium of Web resources for teaching mathematics.
Strengths: Lots of resources recommended for web-based teaching, not just in the calculus.
Weaknesses: Not really a weakness, but the site does need a lot of searching but may well reward the educator with useable resources for classroom use.

URL: http://www.math.psu.edu/dna/graphics.html
Title: Graphics for the Calculus Classroom
Audience: Teachers of mathematics
Level: Suitable for first year undergraduates
Content: A graphics resource intended for teachers
Strengths: Very useful graphics many of which are suitable for immediate inclusion in lessons, tutorials or lectures. Worth a visit.
Weaknesses: None really, this site is offered as a resource for others to use as they see fit.

Group 5 - Foundation Mathematics

URL: http://ole.blc.edu/~rbuelow/FM/WEB1.htm
Title: Foundations of Mathematics.
Audience: Pre-university and first year students
Level: Foundation
Content: This course is intended to serve as a general education course in mathematics and meet the needs for students pursuing Elementary Education. Topics include sets, logic, inductive and deductive reasoning, numeration systems, geometry, probability, and statistics.
Strengths: Interesting site and serves as an example of how such sites can be constructed.
Weaknesses: Non-interactive

URL: http://www.mathmistakes.com/
Title: Foundations of Mathematics.
Audience: All levels
Level: Foundation
Content: This is a list of mathematical mistakes made over and over by advertisers, the media, reporters, politicians, activists, and in general many non-math people.
Strengths: General interest
Weaknesses: Rather thin

URL: http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html
Title: History of Mathematics
Audience: All years
Level: All levels
Content: General history of mathematics
Strengths: Wide ranging and informative
Weaknesses: Non-interactive

URL: http://www.earlham.edu/~peters/courses/logsys/glossary.htm
Title: Discrete Mathematics
Audience: Undergraduate students
Level: First year students

Content: This glossary is limited to basic set theory, basic recursive function theory, two branches of logic (truth-functional propositional logic and first-order predicate logic) and their metatheory.
Strengths: What is there is good
Weaknesses: Non-interactive

URL: http://pass.maths.org.uk/
Title: N/A
Audience: General
Level: All levels
Content: Informative site covering a variety of topics
Strengths: Very attractive design and still being developed
Weaknesses: Little interaction

URL: http://www.shodor.org/master/gnuplot/software/
Title: MASTER: Modelling and Simulation Tools for Education Reform
Audience: Mathematical modellers
Level: All levels
Content: For interactive models and curricular materials, the Foundation offers its "MASTER: Modeling and Simulation Tools for Education Reform" resources. Some of the interactive models include a galaxy simulation, simulated annealing, biomedical and environmental models, and a simulation of Edgar Allen Poe's "Pit and the Pendulum".
Strengths: Interesting collection of possibilities.
Weaknesses: Cost is not specified

URL: http://www.mathsoft.org/mathematics.htm
Title: Foundation Mathematics
Audience: School and College
Level: Foundation
Content: Starting page linking to interactive resources.
Strengths: Good interaction
Weaknesses: Small content

Discussion

An overwhelming impression gained by the members of this Working Group is of the diversity of web sites related to the teaching of mathematics. They range from cases where lecturers have simply placed their notes for a lecture module on the Web, to sites specifically designed as self-contained distance-learning modules with skilfully designed interactive features. Some sites appear not to be aimed at any particular audience at all, while other sites are simply reference lists of other resources available on the Web.

There is also a wide range of quality in the sites visited, with some contributing nothing that could not be done better in print, while others make full use of the potential offered by current internet technology. Many sites are incomplete or under construction.

This observed diversity and range of quality emphasises the importance of developing the skills of efficient searching. Even so, lecturers seeking to use the Internet to enhance their mathematics teaching need to be prepared to spend considerable time and effort searching for resources which fit their requirements - they may in some cases find nothing suitable and so conclude that the best option is to write new materials themselves. A further issue that needs to be considered is that many of the best teaching materials on the Web are designed to be used in conjunction with other technology, e.g. symbolic manipulation packages, so the local availability of such packages needs to be checked.

One notable omission from most of the sites visited was of facilities for the assessment of students' work. It is suggested that while it is very useful to have learning material on-line, the learning process which we require our students to undergo can never be complete without thorough testing procedures in place. Such procedures should enable the students to practice applications of the theory that are taught on-line (or elsewhere) in order that the twin essentials of competence and confidence be developed on an on-going basis. In this phase of learning it is easily argued that feedback is an essential part of the learning process. A reliance on simple messages such as "Well done!" or "That's right, now try the next question" do not provide any help of a mathematical nature, indeed there is anecdotal evidence to suggest that many students find the repeated use of such patronising phrases very off-putting even to the extent that the learning process is impeded. We suggest that what is required is the development and implementation of mathematical feedback systems which are intended to be a genuine help to those who find the subject difficult. We realise that this is not a trivial problem but believe that if the Web is to ever reach its potential as a means of delivering mathematics to students with limited access to staff then the problems inherent in both self-testing and grade-testing must be addressed and overcome.

It is the experience of those responsible for this report that few Web sites address the problem of how best to use them in the teaching situation. It is also generally acknowledged that sites come and go without warning. This places the learner in an invidious position if such a site(s) is used by a teacher as a core delivery agent. With this in mind, and recognising that there is good, well thought-out material waiting to be used, we suggest that initially course designers might consider the following model in which the web is used to enrich the student experience rather than deliver the core learning material. Essentially, a "sheaf" of web addresses may be brought to the attention of the student at appropriate points in the learning material as shown in figure 1 below.

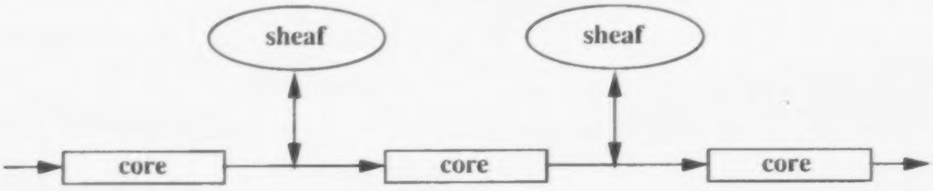

Figure 1 - The Sheaf Model of Learning Material Delivery

The members of the sheaf could be chosen, for example, because they contain particularly relevant graphics, animations, worked examples or quiz questions that the core material does not contain. This places an additional burden on teaching staff since the possible uses of the members of each sheaf will need to be carefully thought out in order to be of real use to the student.

However, the Web is still a young and rapidly expanding technology, and it is anticipated that this and other gaps in provision are likely to filled in the not too distant future.

Conclusions

Potential uses of the Internet as a tool in the teaching of mathematics range from simply giving the students a list of URLs of sites of interest to a total delivery consisting of lecture notes, access monitoring and assessment. However, the important word is "Potential", and it was felt by the group that any description of how to use the Internet would be naive. Articles purporting to describe the use of the Internet very often conclude in a wish list for facilities that can be seen to be useful but either do not yet exist or are only in the stages of primitive development.

Instead, the group decided that a 'starter list' of sites that can be investigated further would be more likely to be of use. It is recommended that these sites be viewed - there is a tremendous amount of material on the Web, much of it worthwhile. Given the diversity in both content and quality of these sites (which only constitute a small sample of what is available), it is clear that teachers will need to formulate a clear idea of how they want to use the web, and will then need to exercise discrimination in choosing appropriate sites. Alternatively, viewing what is already available may provide inspiration to create new materials to fulfil requirements not met by existing web sites. In all cases, whether using existing materials or creating new ones, we emphasise the need to ensure adequate resource provision for students, as detailed in the Introduction.

Supporting the professional development of mathematics lecturers

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Introduction

Every profession requires the continuing development of its practitioners, and, with the advent of the Institute for Learning and Teaching (ILT), universities in general and mathematics departments in particular are being asked to focus attention more proactively on the professional development of their staff *as teachers*. Quality issues regarding teaching, as well as research, are driving many developments in higher education, and the accreditation of professional development activities is presently high on the national agenda. Membership of the ILT currently carries with it a requirement to engage in continuous professional development, specifically a minimum of two days per year. Inevitably Heads of Department and those managing budgets are concerned about the cost implication of this blanket commitment. However, many lecturers are aware of the value of spending time focused on developing their teaching skills. Increasing awareness of the public accountability of universities means that lecturers should be prepared to demonstrate that they have a professional approach to *all* aspects of their work.

The model of professional development which has evolved in many institutions is one of a centrally provided 'staff development unit' which offers one-off generic support sessions. Such sessions can be of great value to many, but the opportunity to engage in subject specific activities can be lost. Mathematics lecturers may find the generic approach inadequate for their needs, and may request professional development based on the collective experience of the mathematics community at large. Inevitably it is difficult for any small group of mathematicians to supply a wide range of professional development needs for themselves; and, while this paper suggests some activities which individual departments might facilitate, we recognise that the professional bodies within mathematics (for example the London Mathematical Society (LMS), the Institute for Mathematics and its Applications (IMA), the Royal Society for Statistics (RSS), the Operational Research Society (ORS)) should be involved in the construction of a national framework for professional development.

In this paper we intend to provide a possible starting point for a conversation between bodies such as the ILT and the professional mathematics societies, university departments, individual lecturers and those groups (including the Undergraduate Mathematics Teaching Conference (UMTC) and the Mathematics, Statistics and OR Support Network (LTSN)) who may be in a position to facilitate professional development programmes. We know that individual universities already support the initial training of lecturers and we are aware of the need for subject-specific provision within that area, but we shall not be considering here this aspect of professional development (in 1996 UMTC addressed it specifically). This paper develops an agenda for Continuing Professional Development (CPD), in the context of a variety of personal models which lecturers bring to the development of their teaching. We include some specific activities to illustrate possible development sessions which could appeal to a wide spectrum of colleagues. We also identify some contexts in which the activities may be implemented, possible forms of delivery, and issues related to the ownership of the activities.

Continuing Professional Development Agendas and personal models

There is a strong perception that there are a number of people in the mathematics community who are opposed to participating in formal CPD for teaching, and this cannot be ignored. There are many other pressures on staff time, not least preparing research papers to satisfy the Research Assessment Exercises (and their own promotion requirements), administration at many levels and the practical issues of course delivery. Two days from a whole working year may seem insignificant, but many staff feel even this is an unwarranted intrusion. We believe that this approach is a short-sighted view of CPD which may well result in the profession as a whole losing public respect. However, in considering the development of CPD in the field of mathematics, we must be aware of this attitude.

We take as a starting point that effective professional development depends more upon action than instruction. That is, whilst "input" is essential for some people at certain times, in general lecturers will develop more from being asked to engage with their own practice and share that practice with others. It is experiences that make a difference.

Lecturers have different expectations of what is meant by CPD and what forms of CPD will be of value to them. Some lectures will request only directed help (they ask 'tell me how to do it') and certainly any framework for CPD should include sessions of this type. However many people will actually get more benefit from sessions in which good practice is shared, or where they can discuss the relative successes or failures of different approaches. Such sessions require skilled and planned facilitation to be effective, and rely heavily on a willingness and ability to reflect on one's own experiences.

In the following section we give a number of examples of CPD activities which could be implemented for the benefit of individuals, departments or clusters of departments. In addition we should be aware of, and encourage, alternative forms of CPD in which individuals engage without the need for frameworks developed by others. For example, we are aware of colleagues who reflect upon their own practice, network informally with others and write informative articles discussing aspects of their teaching. We would see this as just as valid a form of personal CPD, which should be recognised by any accreditation system.

Specific Activities

In the following five subsections we give outline activities which could be developed in different contexts. They are not intended to form an exhaustive list, nor do they illustrate all the different styles of CPD - they are intended as examples only. They are intended to show some possible implementations of professional development at a departmental or cluster level, which could well be suitable for accreditation in a formal scheme. Sessions of the type described could be conducted within departments (with or without an external facilitator) or could be conducted on a regional basis, with the involvement of interested individuals from clusters of institutions.

Example 1: Focusing on facilitating the learning of specific syllabus components

Rationale In some departments colleagues frequently collaborate in order to review the content of their syllabi (and related matters such as the inclusion of key skills), but it is usually left for individual lecturers to review independently their own methods of syllabus delivery. The following example of staff development events both encourages a sharing of current experiences and also initiates creative thought within a department relating to the development of possible new modes of delivery.

Activity The proposed format is that pairs or small groups of colleagues should be asked to focus on specific limited topics within a syllabus and attempt to devise 4 (say) different ways of enabling students to learn the subject. In some cases it might be productive to focus on specific aspects of the delivery of the content. Examples worth considering might include the following

- initiating the topic
- identifying underlying student activities
- identifying opportunities for integrating the use of Information Technology.

Outcomes The expectation is that participants may benefit directly from collaboration with others on the teaching of topics within their teaching portfolio; they can also benefit by extending the ideas discussed to other areas of their teaching.

Example 2 Giving a mathematical flavour to topics which are often viewed as generic from a staff development viewpoint

Rationale It is frequently the case that practising mathematics lecturers consider available staff development events (for topics not perceived to be subject specific) to be unsuitable for their needs; this is because of the symbolic, logical and heirarchical nature of the subject, and the particular responses and needs of students. Thus there is a demand for staff development events facilitated by mathematicians themselves, making use of material and resources familiar to mathematics lecturers. Events belonging to this category of staff development activity would be led by a facilitator who, ideally, would be able to input findings from subject-relevant pedagogic research.

Activity There are many topics where consideration of the mathematical flavour can provide invaluable CPD; learning from existing practice (including bad practice) will typically be one dimension of the event. Relevant topics that can be examined and discussed include:

- writing weekly mathematics worksheets/problem sheets (including layout)
- working with students in small groups
- the flexible use of lecture time with large student groups
- the use of printed materials (including textbooks)
- encouraging students to work in co-operative groups
- the pace and coverage of syllabus content
- peer assessment: observation and feedback
- different ways of assessing mathematics
- the effective incorporation of IT into mathematics learning

Outcomes Having engaged in such activities, participants will have a sense of ownership in having helped generate a set of subject-relevant guiding principles relating to mathematics teaching.

Example 3: Mistakes in Mathematics

Rationale Within mathematics, as in other subjects, we often look to find out what people 'know' or 'understand'. However, much information that is revealing about learning processes can be gathered by studying the 'misunderstandings' that people have and the 'mistakes' that people make. It can be thought that mistakes merely indicate something being 'just wrong', maybe indicating a lack of understanding. However a cooperative examination of mistakes can give insights into inappropriate models being constructed, with students using a wrongly-formed model of a theory; similar mistakes by a group of students can be an indication that a teaching approach should be modified.

Activity In conducting a session, a facilitator can provide copies of work containing 'perennial mistakes' and participants will be invited to bring their own examples of student errors for analysis. During the exercise the participants will

- seek to understand any misconceptions involved
- propose a fruitful response from the teacher to the student
- consider ways that the teacher might modify the approach to avoid the misconception.

As an example of a 'perennial mistake', many of us will have met a student writing "$(a + b)^2 = a^2 + b^2$" as an algebraic 'identity'. In this case the student may be applying the idea of a 'universal linear structure' to the symbols. A lecturer might respond by inviting the student to try numerical examples, but may generate more insight by examining the area of a square with side (a + b) or inviting the expansion of the expression (a + b)(a + b).

Outcomes The examination of common mistakes can be very useful. Lecturers will feel that they are not isolated in their experiences. The session can help them identify topics that are genuinely difficult to understand, topics where students find the lecturer's own model not easy to follow, and situations where an alternative approach can be beneficial (they may wish then to modify the course).

Example 4: Modes of representation - Numerical, Graphical, Symbolic

Rationale In learning mathematics, or working with mathematical ideas, there are often several different modes of representation possible: numerical, graphical or symbolic. Sometimes there is an agreed 'best mode' for a given topic, but many of us respond well if offered a variety - it helps us to consolidate our knowledge. There are many occasions when mathematical ideas can be presented and examined using a several modes; this aids the students' development and understanding. It is a valuable exercise to consider how topics can be introduced and problems examined using a variety of the modes available.

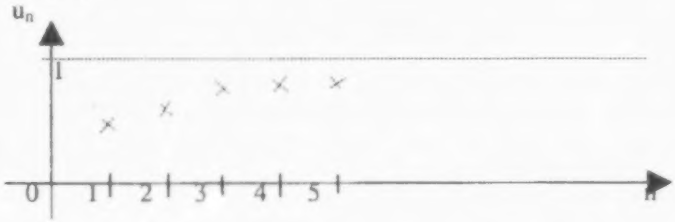
Activity In this type of CPD session a facilitator will encourage participants to consider particular mathematical concepts or topics or problems, and for each of them to construct modes of representation of all the three types (numerical, graphical, symbolic). This can be especially challenging, and useful, where they are commonly represented in only one or two of these ways.

Specific areas where work has already been done on this could be discussed initially. The Calculus Reform Movement (1) in the United States developed the idea that concepts in calculus could mostly be represented in all three ways. Numerical aspects of differentiation and velocity can be introduced by the distances covered in various time intervals, which leads to the idea of average and then instantaneous velocity. Similarly readings can taken from a distance - time graph, from which averaged velocities over different time periods can be found, and also instantaneous velocities (by constructing tangents). Where there is an equation connecting distance and time then an algebraic study of the topic is possible.

The topic of 'Sequences' also allows the application of such a treatment. For example the sequence which commences (numerically) as 1/2, 2/3, 3/4, 4/5, ...
can also be described symbolically using the formula for its nth term, namely

"$u_n = n/(n+1)$"

or illustrated by a graph:



There are other specific examples which can generate discussion about the modes of representation, including:-

- Give a variety of evidence of a positive solution of x = 2sin x between x=π/2 and x=π
- Give a variety of evidence that for x > 0, limit(sin x / x) exists
- Use graphics to exhibit different arrangements of the series
  1 - 1/2 + 1/3 -1/4 + 1/5 - 1/6 ...
- Illustrate the partial sums of the power series for sin x
- Illustrate that a/b ŧħ (a+c)/(b+d) ŧħ c/d, for a,b,c,d □ N and a/b ŧħ c/d.

Outcomes Students learn in different ways, so an appreciation of all three modes of representation will contribute to the quality of support that a lecturer can give to a student.
Example 5: What is 'understanding' in mathematics?

Rationale This example of a CPD session is based on the recognition of the four 'qualities of learning' in mathematics; consideration of these will deepen a lecturer's appreciation of how students learn, and help to inform their own practice when teaching. The four qualities (adapted from Bloom's "Taxonomy of Educational Objectives" (2)) all have a place in the learning of mathematics; the non-hierarchical list is:

- recall of facts (e.g. formulae, definitions), ie knowing *that*
- algorithmic learning, ie knowing *how*
- conceptual learning, ie knowing *why*
- problem-solving procedures (heuristics for dealing with situations that cannot be handled algorithmically)

A lecturer is concerned with teaching and learning, so 'understanding' is a key concern. The word 'understanding' itself can be associated with the conceptual learning (and is a pre-requisite for choosing and applying problem-solving procedures) but also in some contexts with the algorithmic learning. Richard Skemp (3) draws a distinction between 'instrumental' understanding and 'relational' understanding; for example, a student may say that they 'understand' integration by parts on the grounds that they can perform it, rather than appreciating the concepts underlying it or the power of the technique.

Activity We envisage this as a half-day CPD session. A facilitator would introduce the session by outlining the four 'qualities of learning' given above, and invite a brief discussion and consideration of examples suggested by participants. The facilitator would then initiate activities and feedback sessions and finally summarise conclusions. Possible activities could include small group or plenary involvement in:

- looking at exam questions (or problem sheets) provided by the facilitator (or participants) and discussing what kinds of learning they seem to be assessing (or aiming to develop).
- discussion on the teaching of a specific topic, and considering whether and how each of the 'qualities of learning' is addressed.
- considering how lecturers could best approach a diagnostic assessment of understanding,
  for example with questions in tutorials? or by responding to student errors?
- discussion (bearing in mind our time constraints) on the priorities for different kinds of learning in different kinds of courses, for example Mathematics Honours degrees vs Mathematics service courses.
- considering what effects (if any) semesterisation has on the development and assessment of problem solving skills.

Outcomes After such experiences within CPD, participants will be better placed to scrutinise (preferably with colleagues) the documentation and assessments for the courses that they teach. They will be alerted to checking that all four qualities of learning will be developed within the course as a whole, to identifying which topics are likely to emphasise which different qualities, and to ensuring that the assessment strategy truly matches the learning objectives.

Conclusions: implementation, delivery and ownership

Who should take responsibility for CPD for mathematics lecturers?

There are many bodies who might see themselves as candidates for this. Certainly the professional mathematical societies, in taking an overview of the mathematicians' work as a whole, should have a large say in the form and implementation of CPD activities related to teaching. However, organisations which are focused specifically on university mathematics teaching already have existing experience of, and commitment to CPD. It is likely, therefore, that any framework developed should look to that experience for its implementation. Clearly, given the genesis of this paper, we see UMTC as a key player in this activity, and the existing annual conference is one form of CPD. The examples in the previous section show that other forms can and should be developed either under the umbrella of UMTC or by one of the many other organisations, for example the LTSN, the TaLUM group ('Teaching and Learning Undergraduate Mathematics') or the Mathematical Association. The mathematical community should also consider inviting contributions and collaboration from academics involved with Mathematics Education research.

Some of the sample activities described might be developed to be implemented as an afternoon session during (or close to) a general mathematics conference. This would ensure that a large group of mathematicians (not all of whom may have teaching as their *main* commitment) gain access to 'teaching CPD'; a wide variety of experiences from radically different university contexts can be shared, and informal CPD networks can form.

While certain individuals will take responsibility for their own CPD, reflecting on their practice, engaging in action research, publishing, etc, clearly some CPD is best provided within a single department (or regional cluster). This might be seen as one of the most cost effective modes. Some of the examples given could come into this category: in which a facilitator (either from outside the department or, if a suitably skilled member exists, from within) initiates and supports the developmental activities.

Specialist CPD (which might focus on a single topic or small group of topics) may not attract sufficient interest in any single department. For this reason, it may be valuable to offer CPD sessions across a group of mathematics departments clustered geographically. In addition, this allows staff with different experiences, different strengths and different difficulties with the same topic to share their ideas and learn how others teach. Inevitably this requires those attending the session to feel able to share their weaknesses as well as their strengths and such sessions will require careful facilitation. We believe the value of these sessions would outweigh this difficulty.

The question of ownership is inevitably bound up with that of finance. All heads of department have access to staff development funds, though they may feel they are insufficient to cover the ILT requirements at present. The variety of delivery modes suggested should allow for finance to come from many different sources. Professional mathematics societies should consider carefully how they might support their membership in this vital activity; departments across clustering universities might collaborate to reduce per-head costs; existing teaching oriented groups might be encouraged to expand and diversify their existing provision and share resources, and the ILT/LTSN might be able to contribute financial support from within their existing fiscal model (particularly in pump-priming activities).

One of the issues we began this paper with was that of how to encourage all lecturers to engage in CPD activities. We have to acknowledge that it is impracticable to assume that any single CPD framework will encompass everyone. The ILT requirement may encourage some people and may put pressure on some other more reluctant participants. Similarly, heads of department may encourage people still further. However, we believe that a 'three-line whip' approach can be self defeating and, while it may increase attendance, it does not necessarily increase goodwill.

We hope that the range of activities we have suggested and the proposal for the ILT, professional mathematics societies and bodies interested in university mathematics teaching to work together to produce a framework will provide the widest possible level of participation.

References

(1) For information concerning the Calculus Reform Movement:
http://forum.swarthmore.edu/mathed/calculus.reform.html

(2) Bloom, B.S. (1956) 'Taxonomy of Educational Objectives in the Cognitive Domain', Longman, London

(3) Skemp, Richard. (1976) 'Relational Understanding and Instrumental Understanding', Mathematics Teaching 77 (pp.20 - 26)

Appendix

List of Delegates

LIST OF DELEGATES

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Conference Programme

UMTC 2000

Programme

Time	Monday 4th Sept	Tuesday 5th Sept	Wednesday 6th Sept	Thursday 7th Sept
9.00 a.m.		Working Groups	Presentations by delegates (Parallel Sessions)	Reading of comments and reporting back Final Plenary Session & review of UMTC 2000 Suggestions for UMTC 2001
10.30		Tea/Coffee	Tea/Coffee	Tea/Coffee
11:00	Registration / Coffee	Working Groups	Each group introduces its report	Organising Committee Meeting
11:30	Registration and Coffee	Working Groups (continued)	Comments on Draft Reports from other working groups.	Organising Committee Meeting
1:00 p.m.	Lunch	Lunch	Lunch	
2:00	First Plenary *Dr. Simon Singh*	Second Plenary *Prof. Robert Burn*	Conference Outing	
3:00	First Plenary (continued)	Second Plenary (continued)	Conference Outing	
3:30	Tea	Tea		
4:00 -5:30	Working Groups	Working Groups	Conference Outing	
6:00 - 7:00	Dinner	Dinner		

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7:00 - 9:00	Working Groups	Draft Reports written and submitted by 9:00 p.m.	Conference Dinner	

The working group briefs

What use might mathematics education research be to university mathematics teachers?

Introduction

There is a long established tradition of research in university mathematics education. Stretching back over 20 years, researchers have explored misconceptions in calculus, analysis, group theory, discrete mathematics, linear algebra, geometry, logic etc. Work has been done in constructing models of how students may come to learn (or fail to learn) concepts in advanced mathematics which appear to explain and predict some of what observers see happening in lecture theatres and tutorials. However, where there has been much work done in translating education research at the primary and (to a lesser extent) the secondary level to classroom practice, one may question how much university lecturers are influenced by research.

There may be a number of problems involved it this.

University lecturers may not be accessing the research. Perhaps because they choose not to do so, because it is not easily available or it is not in an appropriate language.

Mathematics education researchers may not be doing research that is appropriate Perhaps they are not asking the right questions, their focus is too theoretical, they are too distant from the role of a university mathematics teacher or they are generalising too readily from school based research.

University administrators may not be giving support for research-based change. Perhaps research-led change is too radical with no smooth path from the current position to the new one. Perhaps the practicalities of how to change are not addressed. Perhaps, even, the research indicates what the problems are but tells the lecturer nothing about the solution.

The group may choose to explore some of the papers provided which have been chosen to reflect a range of research within university mathematics education and ask how these papers might address the needs of the university mathematics teacher.

The Remit

Delegates intending to join this working group may choose to bring along research papers which have influenced their own teaching, or explain how they have modified their work as a result of some output under the heading of university mathematics education research.

- They might ask what addition work might need to be done to these papers to make them of more use to university mathematics teachers.

- The group may wish to consider how they might inform educational researchers of their needs and the form in which they would like the 'results'.

- The group might consider which areas of university mathematics teaching and learning are in need of investigation and how that investigation should be carried out.

- They may ask what kind of research might university mathematics teachers become involved in themselves to better inform their teaching and how mathematicians and mathematics educators might work more closely in producing meaningful and practical research.

References

Leron, U., Hazzan, O and Zazkis, R. (1995), "Learning group isomorphism: a crossroads of many concepts", *Educational Studies in Mathematics*, 29(2), 153-74

Hillel, J. and Sierpinska, A. (1994) "On one persistant mistake in linear algebra" in da Ponte, J.P. and Matos, J.F. (Eds.) *Proceedings of the 18th International Conference for the Psychology of Mathematics Education*, Vol 3, 65-72.

Tall, D.O. and Vinner, S. (1981) "Concept image and concept definition in mathematics with particular reference to limits and continuity" *Educational Studies in Mathematics*, 12(1), 151-169.

Berry, J.S and Houston, S.K, (1995) "Students Using Posters as a Means of Communication and Assessment", *Educational Studies in Mathematics*, 29, 21-27

D'Inverno, R. (1993) "On the success of a self-paced course" *International Journal of Mathematics Education in Science and Technology*, 24(5), 727-739.

Attracting students to mathematics

Introduction

Mathematics is, numerically, one of the most popular A-levels and yet the number of students going on to study mathematics at university is falling. At some Higher Education Institutions mathematics is a threatened or indeed extinct species of study. What actions can staff in mathematics departments take to reverse this trend? How can we take account of the changing nature of sixth form curricula, particularly current discussions on the structure of A/AS levels, in planning and promoting mathematics courses in Higher Education? Could the increasingly varied mathematical background of potential students be seen as a width of experience to be exploited by recruitment initiatives? Why do mathematics degrees not attract able A-level students in greater numbers?

Many A-level students study mathematics because it supports their particular A-level interest - in physics, engineering, business studies et cetera. Are potential mathematicians lost because most sixth-formers see mathematics as a service subject and have no thought of mathematics for mathematics sake? If so, do we need to attract students to mathematics at 15/16 years, before they have started to follow other career paths and possibly developed a mind-set about mathematics?

It has been suggested that students cannot immediately 'see' where a mathematics degree leads and a working group at UMTC '98 addressed the particular problem of 'What is a mathematics degree for?' Their report has implications both for the structure of mathematics courses and the promotion of mathematics degrees to potential students.

The later careers of those who have opted for mathematics seem for the most part to be unknown to A-level students, perhaps because the width of options available to mathematics graduates means there is no stereotypical career path. Could this inability to stereotype mathematicians be a 'selling point' in a bid to attract more students? *Should we promote a range of 'role models'*? Are there well known, successful people, other than mathematicians, who have mathematics degrees?

What do potential students, (at a range of ages) know about university mathematics. Are we being stuck with an out-dated image? What areas of university mathematics can we use to convey a sense of the practical, imaginative and elegant aspects of our subject?

Do we need to ensure that people realise that 'maths' covers a wide spectrum of topics - pure, applied, methods, modelling, numerical methods, statistics, operational research, computing, et cetera; and that areas such as management science, product design or engineering need mathematicians to use a sophisticated blend of these topics to make numerate business decisions.

The Remit

- Identify 'selling points' which could be exploited to attract students to mathematics degrees.
- Outline strategies for promoting university mathematics to potential students.
- Identify appropriate stages of secondary education for promoting mathematics degrees.
- Suggest opportunities for promoting mathematics which may arise from changes in A/AS level mathematics.

References

1) LMS leaflet - 'What to do with a Mathematics degree'
2) Mike Howkins - DMU internal document - B Sc Mathematics Careers Profile

Innovations in teaching Discrete Mathematics

Introduction

Discrete mathematics modules are a feature of most undergraduate mathematics degree programmes, and have certainly featured in mathematics 'service' teaching for computer scientists and engineers. Such modules often start with such topics as sets, logic, Boolean algebra, functions and relations, etc. Many textbooks exist (see references for examples) that cover such topics. Some of these topics can be found on A-level syllabuses. One might also include such topics as graph theory, combinatorics, operational research and other similar topics often described as 'finite mathematics' or perhaps 'decision mathematics'.The applications of discrete mathematics to engineering and business seem to be increasing, with financial mathematics being one obvious growth area. Programme designers have an increasingly difficult task in ensuring that content is up-to-date, of the appropriate standard and relevant. The inclusion of elements of discrete mathematics, perhaps at the expense of more traditional continuous mathematics teaching, can cause much debate.

The use (or not) of technologies such as computer-based learning software (including that on the Web) or computer algebra systems is also a contentious issue in the teaching of (for example) calculus and the same issues can arise in the teaching of elements of discrete mathematics, although perhaps the impact in this area of mathematics is not so pronounced. Thus the inclusion of discrete mathematics can raise pedagogical issues as well as curriculum/programme development issues.

In summary, the questions posed are:

- What discrete mathematics topics should be taught?
- What innovative ways of teaching such topics have been or ought to be tried?
- How can technology be most appropriately used in the teaching of discrete mathematics?

The Remit

The working group is asked to consider innovative features that could figure in the design of a discrete mathematics module (or modules) either forming a component of a mathematics undergraduate degree programme or serving as a 'service' module(s). The following questions could form the basis for discussion.

Curriculum/Programme design issues

- What topics would a single module 'discrete mathematics' cover? Should all mathematics undergraduate degrees have such a module (or modules?) in year 1?

- What is a reasonable balance between 'continuous' and 'discrete' mathematics modules in a single honours mathematics undergraduate degree programme? Ought modules to contain either discrete or continuous mathematics, or might both be tackled within the same module? For example, do we teach differential equations and difference equations in the same module? If the balance favours too much continuous work then what should be removed?

- Assuming a broad definition of the term 'discrete mathematics', should students be introduced to elements of topics such as graph theory, number theory, combinatorics, operational research, etc. as core activities or are these best developed as separate modules leading to final year **option** choices?

Teaching & learning issues

- To what extent should applications such as financial mathematics, computer network analysis, etc. be paramount in the teaching of discrete mathematics? What applications are important, both now and in the foreseeable future, for a working mathematician to be familiar with, and which are best left to dedicated 'service' modules? Does 'fun maths', such as the study of the mathematics of puzzles and games, have a place here?

- What impact are technologies such as computer-based learning materials (e.g. WWW, MATHWISE), computer algebra systems, etc. having now on the teaching, learning and assessment of discrete mathematics (including service teaching)? How are such technologies being used and with what result? What impact might or should they have in the future?

References

Biggs, N. L. , Discrete Mathematics. Oxford University Press. ISBN 0-19-853426-4
Garnier & Taylor, Discrete Mathematics for New Technology.
Gerstang J.L., Mathematical Structures for Computer Science. Freeman. ISBN 0-7167-1802-2
Molluzzo J. C. & Buckley F., A First Course in Discrete Mathematics. Wadsworth Press. ISBN 0-534-05310-6
Piff M. Discrete Mathematics: An Introduction for Software Engineers. CUP 1991.
Rosen K.H. Discrete mathematics & its Applications. McGraw Hill 1999.
Skaricus R., & Robinson, W.B., Discrete Mathematics with Computer Science Applications. Addison-Wesley 1986.
Truss J.K. , Discrete Mathematics for Computer Scientists. Addison-Wesley, 1991

Davison R. & Cochrane L., Decision Mathematics (vols 1 & 2). Cranfield Press. ISBN 1-871315-30-1 and 1-871315-32-8
Goldstein L. J. , Schneider D.I., Siegal M.J., Finite Mathematics & its Applications. Prentice Hall. ISBN 0-13-082778-9

Hagelgans N.L., 'Constructing the concepts of Discrete mathematics with DERIVE'. Int. J. of Computer Algebra in Mathematics Education (formerly Int. DERIVE Journal) Vol 2 pp115-136. (1995)

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MATHWISE self-study material, Discrete Mathematics module. Contact: Mathwise User Group. School of Mathematics & Statistics, University of Birmingham. http://www.bham.ac.uk/ctimath/mwug/mwug.htm

and many more........

Use of the internet in teaching mathematics

Introduction

(It is hoped that delegates participating in this group will have a variety of levels of experience in the area, from very little to a great deal; the emphasis will be on discussing how best to exploit the internet, rather than on learning how to use it.)

The Internet has made a large impact on the way mathematicians work at universities, assisting in collaboration, and providing (relatively) easy access to reference materials [see 1]. It also provides a way of distributing research materials.

For student learning, the question is how to exploit this in a useful way. Some academics use the web to manage their courses and monitor their students. For some, it is sufficient simply to put HTML (or even just PostScript) versions of their lecture notes on the internet, providing little more than a direct alternative to a book. However, ideally we want to make use of the interactive multimedia and hypertext facilities offered by the Internet.

Producing HTML, and its more advanced developments (XML, MML) is becoming ever easier, but mathematics still is not completely integrated yet. Some computer algebra packages will produce html versions of their worksheets, and even create animated gifs in an easy manner. Java and other distributed programming languages offer ways to distribute packages without having to have them all installed and maintained locally.

The internet offers options for how to collaborate with others and how to work with students, for example creating and sharing courseware between universities [e.g. 2], or using conference software to allow the teaching of more specialised topics to small groups of students at a number of institutions (as is done in Wales). Appropriate use of communications packages may be a way to promote group working, e.g. using email, or more structured environments such as First Class.

Some questions that are worth asking are:

- What *tools* do we need to make the most of the Internet? (Document preparation, interactive packages, on-line questions, on-line exams, mathematics packages, communications, newsgroups, email lists, chat rooms?)
- What are the best ways to *use* the Internet?

A final point, currently the cause of much discussion, is that as a reference source, the internet is a massive resource, and simply encouraging students to use this in gathering materials for projects is likely to lead to positive results. However there can be a downside to this. The ease of cutting and pasting, means that projects and essays become more open to plagiarism, and the relative ease of getting assistance with problems means that any assessed course-work needs to be monitored.

The Remit

The working group is asked to consider the impact, both current and potential, of the world-wide web on the ways in which mathematics courses can be managed, taught and assessed, in order to create a more interesting environment for students, and hopefully to promote the understanding of concepts and good learning habits. Here are some particular questions that they may like to use as a basis for discussion.

- What examples are there of good practice in the use of the web in mathematics courses?
- What are the implications of extensive web use for the way that students learn?
- What tools are required to allow the best exploitation of the web for mathematics education.
- If courses and/or materials are to be shared, what mechanism needs to be put into place to ensure reliability of essential web sites and technology?
- How can best practice in use of the web be identified and spread?
- What are the short-term and long-term support needs for staff embarking on web exploitation?

References

1. The CTI Mathematics Home Page provides an excellent gateway to other sites that show the current state of mathematics on the web. http://www.bham.ac.uk/ctimath/index.html

2. A good illustration of a co-operative project in the USA is the Connected Curriculum Project, based at Duke, Montana and CalPoly http://www.math.duke.edu/education/ccp/index.html

Supporting the professional development of mathematics lecturers

Introduction

The level of attention given to the professional development of lecturers has increased in recent years, with the publication of the Dearing Report [1], teaching quality assessments, the advent of the Institute for Learning and Teaching [2] with its associated Learning and Teaching Support Networks [3], and so on. Lecturers, for instance, are now required to demonstrate a commitment to continuing professional development as a condition of continuing membership of the ILT.

While it is possible to see this agenda as driven by factors that are external to the concerns of lecturers, it can also be argued that professionalism itself demands that lecturers pay attention to their own development. This is particularly true in regard to the teaching of students — an area which can easily be overlooked in comparison to research. Furthermore, the need to attract students to the study of mathematics will also demand the highest of professional standards, especially as market forces increasingly dominate higher education. Furthermore, the study of mathematics in higher education is now changing at a fast pace, whether as a result of new technology, a more diverse student population, changes in mathematics in schools, or otherwise. Such change ensures that professional development is a necessity rather than an externally imposed burden.

In such a climate, it is essential that attention be given to ensuring both that an appropriate range of opportunities for professional development is open to mathematics lecturers, and that lecturers take advantage of what is available. This is particularly true in a distinctive discipline such as mathematics in which the nature of the subject matter plays an important role in teaching and learning.

The Remit

In the light of these issues, the group is asked to provide a report that outlines a way forward for the professional development of mathematics lecturers. Given that the initial training of lecturers has already been discussed at a recent conference [4] and that plans are in train for a Learning and Teaching Support Network in Mathematics, Statistics and Operational Research to address this issue [3], the group may prefer to focus on the continuing professional development of mathematics lecturers. In doing so, you may wish to consider the following questions:

- What good practice already exists in supporting professional development?
- What opportunities for professional development should be provided for mathematics' lecturers? (You may wish to consider the following: development of a road show to visit departments; the place of research into the pedagogy of undergraduate mathematics; training in the use of technology.)

- Who should provide the professional development and how should it be co-ordinated? (You may wish to consider the role of the LTSN, LMS, MA, IMA, RSS, TaLUM, and other bodies.)
- What is the role of UMTC in supporting professional development?
- How should national, institutional, departmental and individual priorities for professional development be reconciled with each other?
- How can we ensure that professional development is not divorced from mathematical content?
- How can mathematics lecturers be encouraged to engage in professional development?

References

[1] National Committee of Inquiry into Higher Education (1997) *Higher Education in the Learning Society* (Middlesex: NCIHE)

[2] Details on the Institute for Learning and Teaching can be accessed at http://www.ilt.ac.uk

[3] Proposal for a learning and teaching Support Network Subject Centre in Mathematics, Statistics and Operational Research, http://www.bham.ac.uk/ctimath/ltsnbid.pdf

[4] Houston K *et al* (1996) *Induction for new lecturers in mathematics*, Proceedings of UMTC 1995, 91-101

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EFF-089 (3/2000)